cuemath

Chapter - 6: Triangles

Exercise 6.1 (Page 122 of Grade 10 NCERT)

Q1. Fill in the blanks using the correct word given in brackets:
(i) All circles are ____________. (congruent, similar)
(ii) All squares are ____________. (similar, congruent)
(iii) All ____________ triangles are similar. (isosceles, equilateral)
(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are _____________ and (b) their corresponding sides are ____________. (equal, proportional)

Difficulty Level : Easy

(i) Reasoning:

As we know that two similar figures have the same shape but not necessarily the same size. (Same size means radii of the circles are equal)

Solution:
Similar.

Since the radii of all the circles are not equal.

(ii) Reasoning:

Same as above (i) same size means sides of the squares are equal.

Solution:
Similar.

Since the sides of the squares are not given equal.

(iii) Reasoning:

All equilateral triangles are similar.

Solution:
Equilateral.

Each angle is an equilateral triangle is 60°.

(iv) Reasoning:

As we know that two polygons of same number of slides are similar if their corresponding angles are equal and all the corresponding sides are in the same ratio or proportion.

Solution:

(a) Equal
(b) Proportional

(a) Since the polygons have same number of sides, we can find each angle using formula $\left(\frac{2n-4}{n}\right)$ right angles. Here 'n' means number of sides of a polygon.

(b) We can verify by comparing corresponding sides.

Related Problems:

Are all similar figures congruent?

Solution:

All congruent figures are similar but all the similar figures need not be congruent.

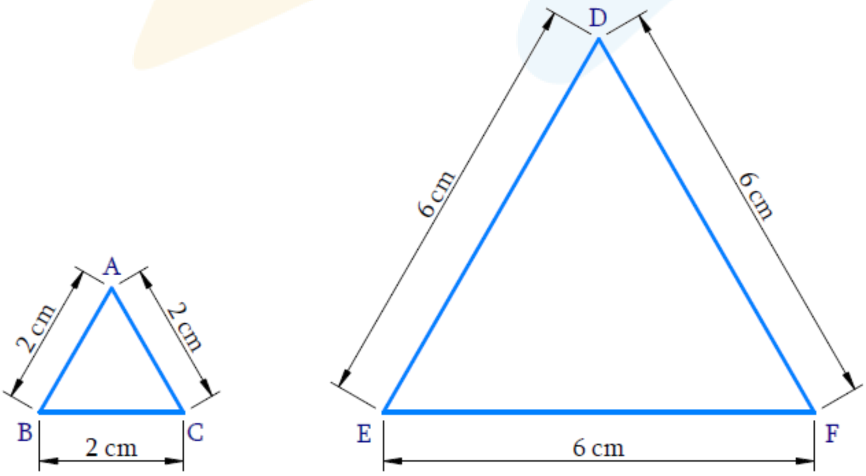
Q2. Give two different examples of pair of

(i) similar figures
(ii) non-similar figures

Difficulty Level : Easy

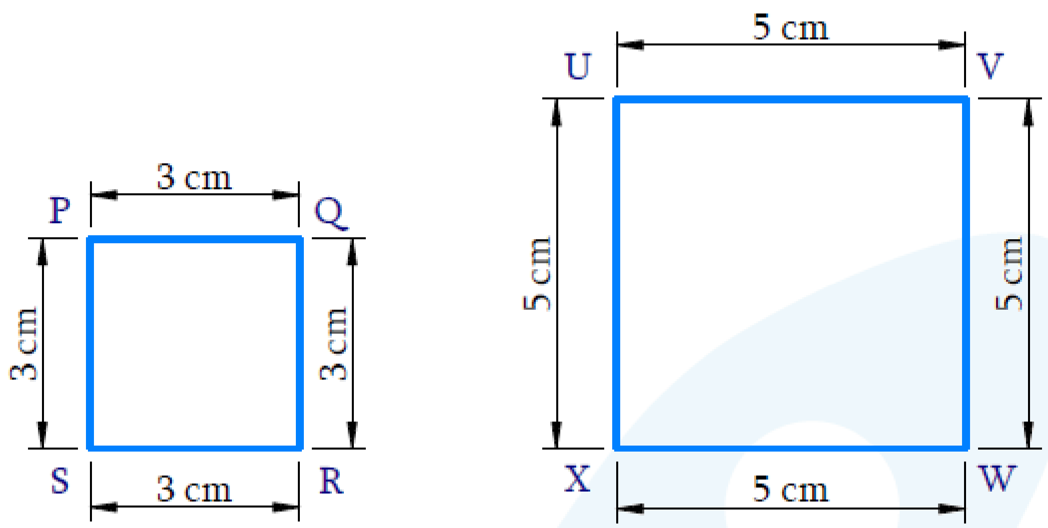
Solution:

(i) Two equilateral triangles of sides 2cms and 6cms are examples for Similar figures.

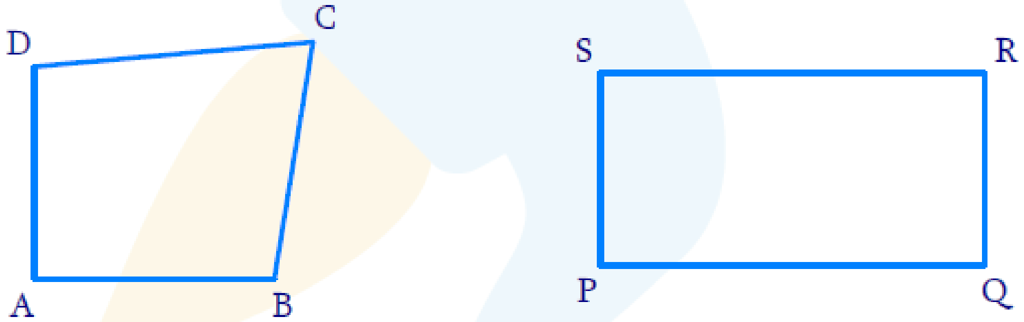


$\Delta ABC \sim \Delta DEF$ (~ is similar to)

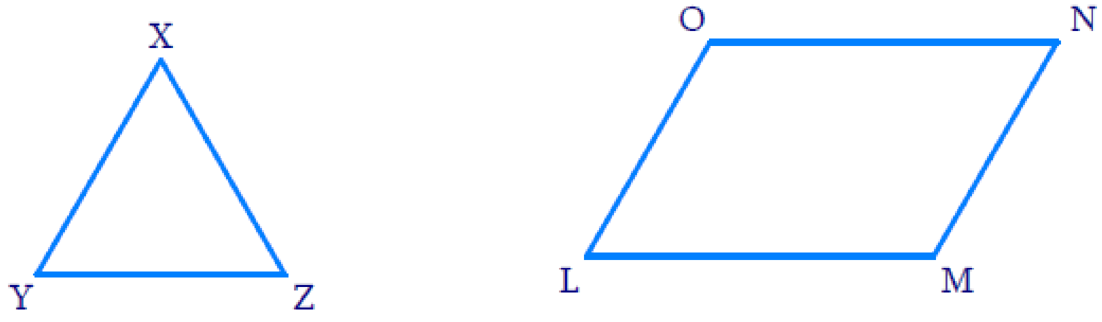
Two squares of sides 3cms and 5cms are examples for Similar figures.



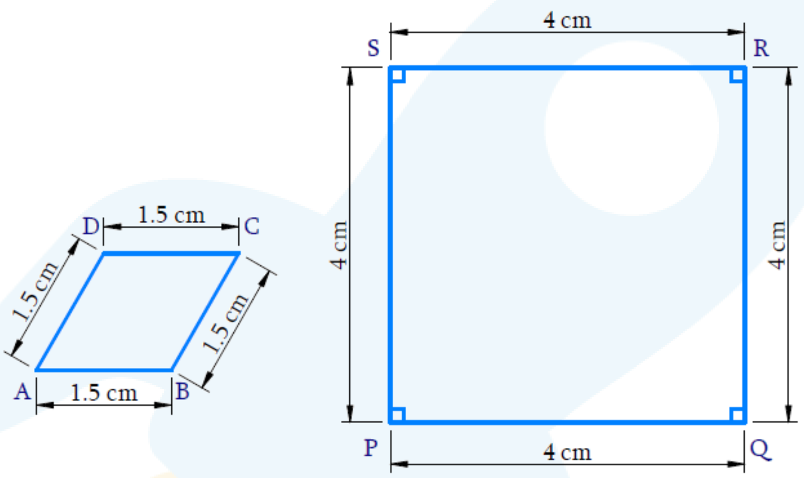
(ii) A quadrilateral and a rectangle are examples for Non-Similar figures.



A triangle and a parallelogram are examples for Non-Similar figures.



Q3. State whether the following quadrilaterals are similar or not:



Difficulty Level : Easy

Reasoning:

Two polygons of the same number of sides are similar, if (i) all the corresponding angles are equal and (ii) all the corresponding sides are in the same ratio (or proportion).

Solution:

ABCD $\nsim$ (is not similar to) PQRS.

In Quadrilateral ABCD and PQRS

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CD}{RS} = \frac{DA}{SP}$$

$\Rightarrow$ Corresponding sides are in proportion

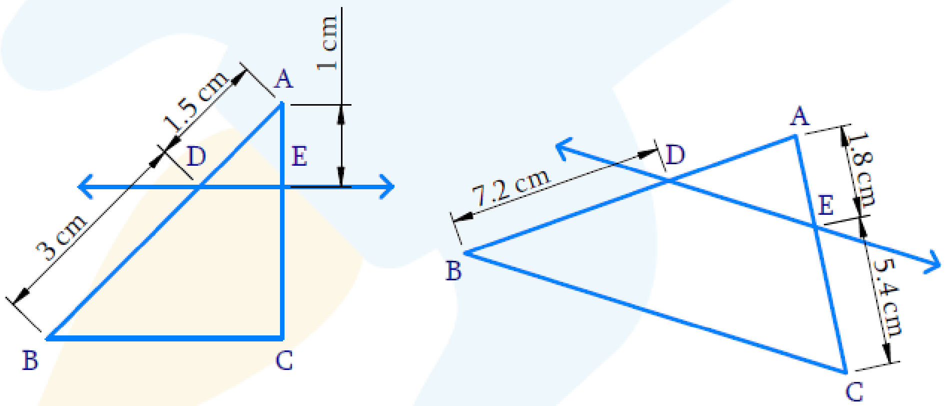
But $\angle A \neq \angle P; \angle B \neq \angle Q$

Corresponding angles are not equal

Therefore, Quadrilateral ABCD $\nsim$ (is not similar to) Quadrilateral PQRS

Exercise 6.2 (Page 128 of Grade 10 NCERT)

Q1. In Fig. 6.17, (i) and (ii), DE || BC. Find EC in (i) and AD in (ii)



Difficulty Level : Medium

Reasoning:

As we all know the basic proportionality theorem (Thales Theorem)(B.P.T)
Two triangles are similar if :
(i) Their corresponding angles are equal
(ii) Their corresponding sides are in the same ratio (or proportion)

Given :
DE || BC

Solution:

(i) In, ΔABC

$$BC \parallel DE$$

In ΔABC & ΔADE

$\angle ABC = \angle ADE$ [$\because$ corresponding angles]

$\angle ACB = \angle AED$ [$\because$ corresponding angles]

$\angle A = \angle A$ common

$\Rightarrow \Delta ABC \sim \Delta ADE$

Since the two triangles are similar, their corresponding sides are in the same ratio.

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{1.5}{3} = \frac{1}{EC}$$

$$EC = \frac{3 \times 1}{1.5}$$

$$EC = 2\text{cm}$$

(ii) Similarly, $\Delta ABC \sim \Delta ADE$

Since the two triangles are similar, their corresponding sides are in the same ratio.

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$AD = \frac{7.2 \times 1.8}{5.4}$$

$$AD = 2.4\text{cm}$$

Q2. E and F are points on the sides PQ and PR respectively of a Δ PQR. For each of the following cases, state whether EF || QR:

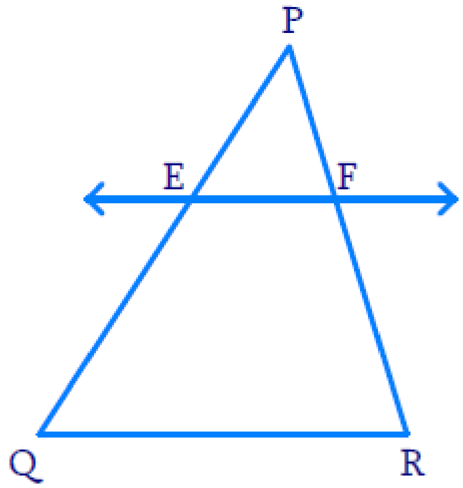
(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm
(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm
(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

Difficulty Level : Easy

(i) Reasoning:

As we know that a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side (converse of BPT)

Solution:



According to converse of BPT

$$\frac{PE}{EQ} = \frac{PF}{FR}$$
$$EF \parallel QR$$
$$\frac{PE}{EQ} = \frac{3.9}{3}$$
$$= 1.3\text{cm}$$
$$\frac{PF}{FR} = \frac{3.6}{2.4}$$
$$= \frac{36}{24}$$
$$= \frac{3}{2}$$
$$= 1.5$$

$$\Rightarrow \frac{PE}{EQ} \neq \frac{PF}{FR}$$

$\Rightarrow$ EF is not parallel to QR.

(ii) Reasoning:

Theorem 6.2 : If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Solution:
According to converse of BPT

$$\frac{PE}{EQ} = \frac{PF}{FR}$$
$$EF \parallel QR$$
$$\frac{PE}{EQ} = \frac{4}{4.5}$$
$$= \frac{8}{9}$$
$$\frac{PF}{FR} = \frac{8}{9}$$
$$\Rightarrow \frac{PE}{EQ} = \frac{PF}{FR}$$
$$\Rightarrow EF \parallel QR$$

(iii) **Reasoning:**

Theorem 6.2: If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Solution:

According to converse of BPT

$$\frac{PE}{EQ} = \frac{PF}{FR}$$
$$\Rightarrow EF \parallel QR$$

PQ = 1.28 cm and PE = 0.18cm

$\Rightarrow$ QE = PQ – PE
= 1.28 – 0.18
= 1.10 cm

PR = 2.56 cm
PF = 0.36 cm

$\Rightarrow$ RF = PR - PF
= 2.56 – 0.36
= 2.20 cm

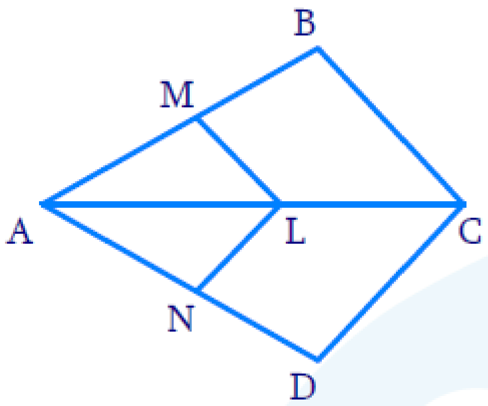
Now,

$$\frac{PE}{EQ} = \frac{0.18cm}{1.10cm}$$
$$= \frac{18}{110}$$
$$\frac{PF}{FR} = \frac{0.36cm}{2.20cm}$$
$$= \frac{36}{220} = \frac{18}{110}$$

$$\Rightarrow EF \parallel QR \left(\because \frac{PE}{EQ} = \frac{PF}{FR} \right)$$

Q3. In Fig. 6.18, if LM || CB and LN || CD, prove that

$$\frac{AM}{AB} = \frac{AN}{AD}$$



Difficulty Level: Medium

Reasoning:

As we know if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Solution:

In ΔABC

$$LM \parallel CB$$

$$\frac{AM}{MB} = \frac{AL}{LC} \text{........... (Eq 1)}$$

In ΔACD

$$LN \parallel CD$$

$$\frac{AN}{DN} = \frac{AL}{LC} \text{............(Eq 2)}$$

From (1) and (2)

$$\frac{AM}{MB} = \frac{AN}{DN}$$

According to Euclid's axiom (1): Things which are equal to the same thing are equal to one another.

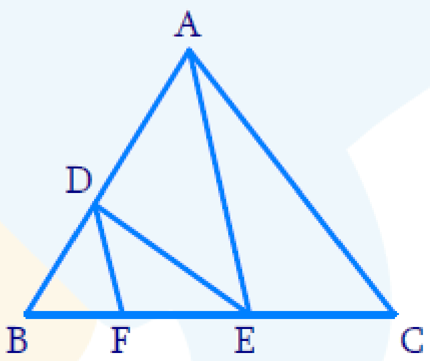
$$\Rightarrow \frac{MB}{AM} = \frac{DN}{AN}$$

Adding 1 on both sides

$$\frac{MB}{AM} + 1 = \frac{DN}{AN} + 1$$
$$\frac{MB + AM}{AM} = \frac{DN + AN}{AN}$$
$$\frac{AB}{AM} = \frac{AD}{AN}$$
$$\frac{AM}{AB} = \frac{AN}{AD}$$

Q4. In Fig. 6.19, DE || AC and DF || AE. Prove that

$$\frac{BF}{FE} = \frac{BE}{EC}$$



Difficulty Level : Medium

Reasoning:

Theorem 6.1 : If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Solution:

In ΔABC

$$\frac{BD}{AD} = \frac{BE}{EC} \quad ....(i)$$

In $\Delta\, ABE$

$DF \parallel AE$

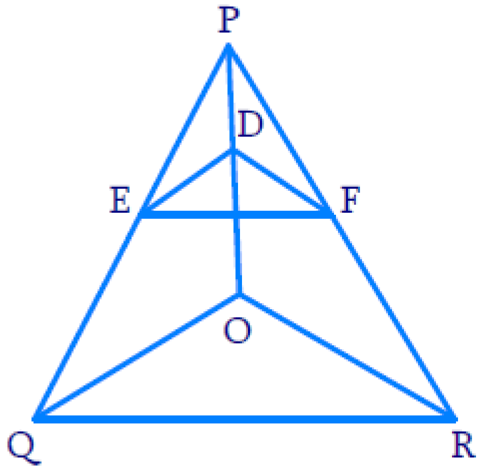
$$\frac{BD}{AD} = \frac{BF}{FE} \quad ....(ii)$$

From (i) and (ii)

$$\frac{BD}{AD} = \frac{BE}{EC} = \frac{BF}{FE}$$

$$\frac{BE}{EC} = \frac{BF}{FE}$$

Q5. In Fig. 6.20, DE || OQ and DF || OR. Show that EF || QR.



Difficulty Level : Medium

Reasoning:

Theorem 6.2: If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Solution:

Using Theorem 6.2

In ΔPOQ

$$DE \parallel OQ \text{ (given)}$$

$$\frac{PE}{EQ} = \frac{PD}{OD} \quad ....(1)$$

In ΔPOR

$$DF \parallel OR\left(given\right)$$

$$\frac{PF}{FR} = \frac{PD}{DO} ....(2)$$

From (1) & (2)

$$\frac{PE}{EQ} = \frac{PF}{FR} = \frac{PD}{DO}$$

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

According to Euclid's axiom (1): Things which are equal to the same thing are equal to one another.

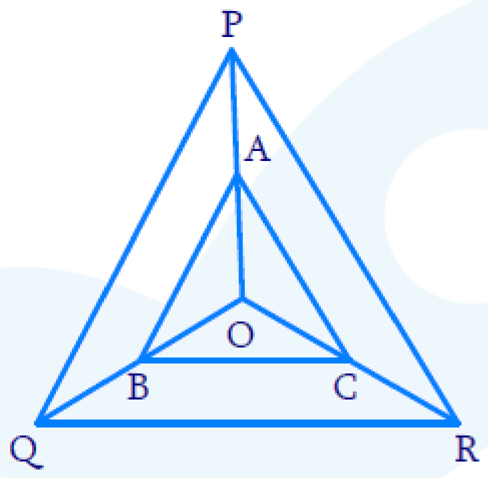
In ΔPQR

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$QR \parallel EF$ (Converse of BPT page no. 126)

According to **Theorem 6.2** : If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Q6. In Fig. 6.21, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.



Difficulty Level : Medium

Reasoning:

According to **Theorem 6.2** : If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Solution:

In ΔOPQ

$AB \parallel PQ$ (given)

$$\frac{OA}{AP} = \frac{OB}{BQ} \quad ....(i)$$

[$\because$ Theorem 6.1 BPT]

Theorem 6.1 : If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

In ΔOPR

$$AC \parallel PQ \text{(given)}$$

$$\frac{OA}{AP} = \frac{OC}{CR} \text{..............(ii)}$$

$$[\because \text{Theorem 6.1 BPT}]$$

Theorem 6.1 : If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

From (i) & (ii)

$$\frac{OA}{AP} = \frac{OB}{BR} = \frac{OC}{CR}$$

According to Euclid's axiom (1): Things which are equal to the same thing are equal to one another.

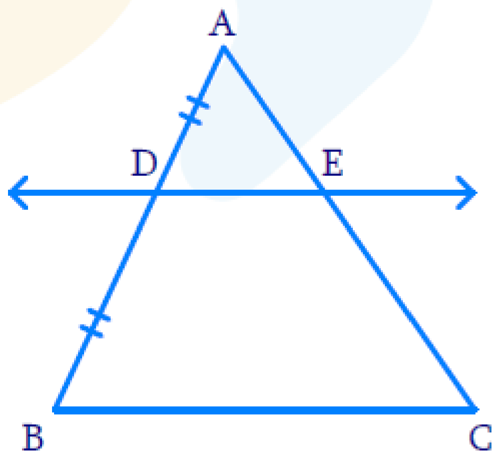
$$\frac{OB}{BQ} = \frac{OC}{CR}$$

Now, In ΔOQR

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

$$BC \parallel QR [\because \text{Theorem 6.2}]$$

Q7. Using Theorem 6.1, prove that a line drawn through the midpoint of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).



Difficulty Level : Medium

Reasoning:

Theorem 6.1 states that "If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio (BPT)".

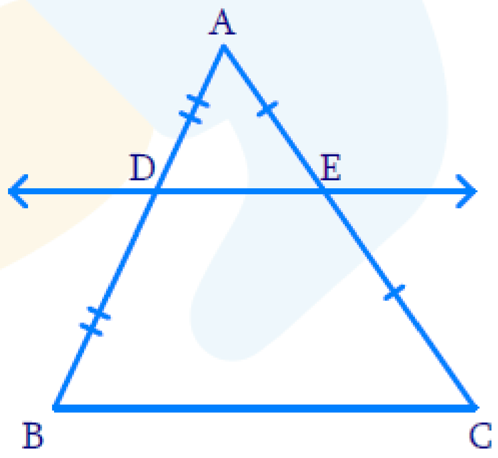
Solution:

In ΔABC , D is the midpoint of AB
AD = BD

$$\frac{AD}{BD} = 1$$
$$DE \parallel BC$$
$$\frac{AE}{EC} = \frac{AD}{BD}$$
$$\frac{AE}{EC} = 1$$

$\Rightarrow$ E is the midpoint of AC.

Q8. Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).



Difficulty Level : Medium

Reasoning:

Theorem 6.2 tells us if a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side. (Converse of BPT)

Solution:

In ΔABC

D is the midpoint of AB

AD = BD

We can write,

$$\frac{AD}{BD} = 1 \text{ ....(i)}$$

E is the midpoint of AC

AE = CE

We can write,

$$\frac{AE}{BE} = 1 \text{.........(ii)}$$

From (i) and (ii)

$$\frac{AD}{BD} = \frac{AE}{BE} = 1[\text{Euclid's axiom}]$$

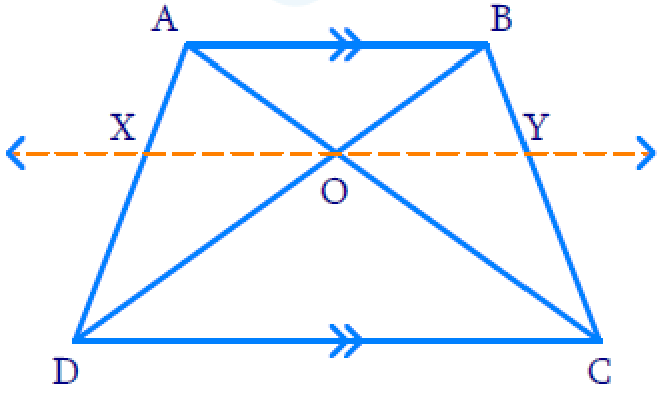
$$\frac{AD}{BD} = \frac{AE}{BE}$$

According to Euclid's axiom (1): Things which are equal to the same thing are equal to one another.

Thus, according to theorem 6.2,

$$DE \parallel BC$$

Q9. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$



Difficulty Level : Medium

Reasoning:
Theorem 6.1 : If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Solution:

In trapezium ABCD

AB||CD and diagonals AC, BD intersect at 'O'

Construct XY||AB and CD through 'O'

In ΔABC

$$OY \parallel AB \left(\because \text{ construction}\right)$$

According to theorem 6.1 (BPT)
Theorem 6.1 : If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

$$\frac{BY}{CY} = \frac{OA}{OC} \text{ ....(I)}$$

In ΔBCD

$$OY \parallel CD$$

$$\frac{BY}{CY} = \frac{OB}{OD} \text{.................}(\text{II})\left[\text{According to BPT}\right]$$

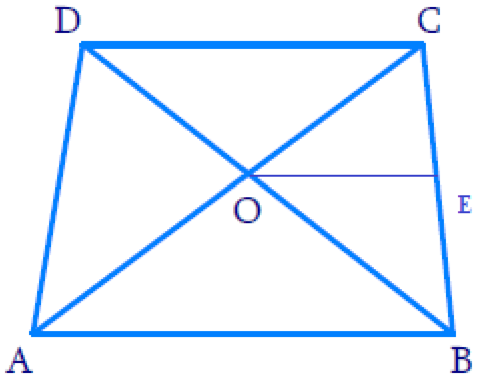
From (I) and (II)

$$\frac{OA}{OC} = \frac{OB}{OD} \left[\text{Euclid's axiom 1 of Grade 9}\right]$$

$$\Rightarrow \frac{OA}{OB} = \frac{OC}{OD}$$

According to Euclid's axiom (1): Things which are equal to the same thing are equal to one another.

Q10. The diagonals of a quadrilateral ABCD intersect each other at the point 'O' such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.



Difficulty Level : Medium

Reasoning:

Theorem 6.1 [BPT] : If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Solution:

In quadrilateral ABCD
Diagonals AC, BD intersect at 'O'

Draw OE||AB

In ΔABC

$$OE \parallel AB$$

$$\Rightarrow \frac{OA}{OC} = \frac{BE}{CE} \,(BPT)\ldots\ldots\ldots\ldots(1)$$

But $\frac{OA}{OB} = \frac{OC}{OD} \,(given)$

$$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}\ldots\ldots\ldots\ldots(2)$$

From (1) and (2)

$$\frac{OB}{OD} = \frac{BE}{CE} \text{ [Euclid's Axiom-1]}$$

According to Euclid's axiom (1): Things which are equal to the same thing are equal to one another.

In ΔBCD

$$\frac{OB}{OD} = \frac{BE}{CE}$$

$$OE \parallel CD$$

$$OE \parallel AB \text{ and } OE \parallel CD$$

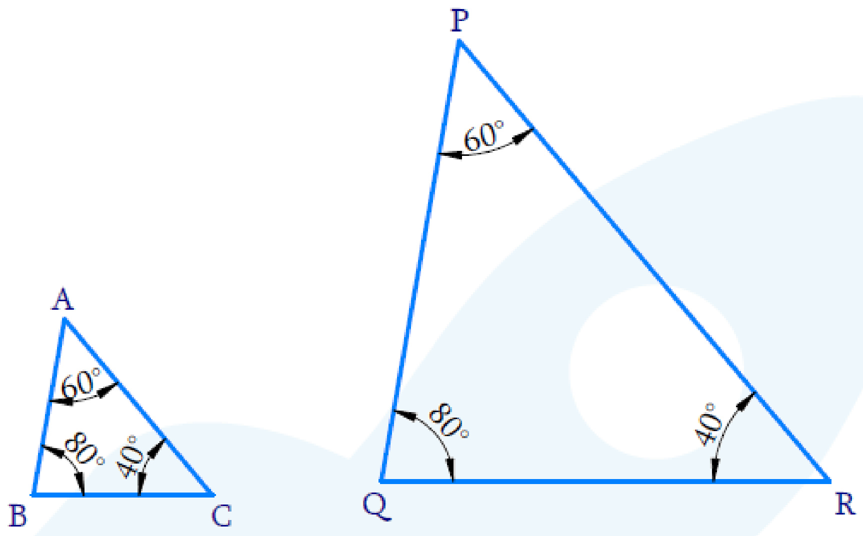
$$\Rightarrow AB \parallel CD$$

$\Rightarrow$ $ABCD$ is a trapezium

Exercise 6.3(Page 138 of Grade 10 NCERT)

Q1. State which pairs of triangles in Fig. 6.34 are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

1)



Difficulty Level: Medium

Reasoning:

Theorem 6.3: If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.

This criterion is referred to as the AAA (Angle–Angle–Angle) criterion of similarity of two triangles.

Solution:

According to AAA criterion

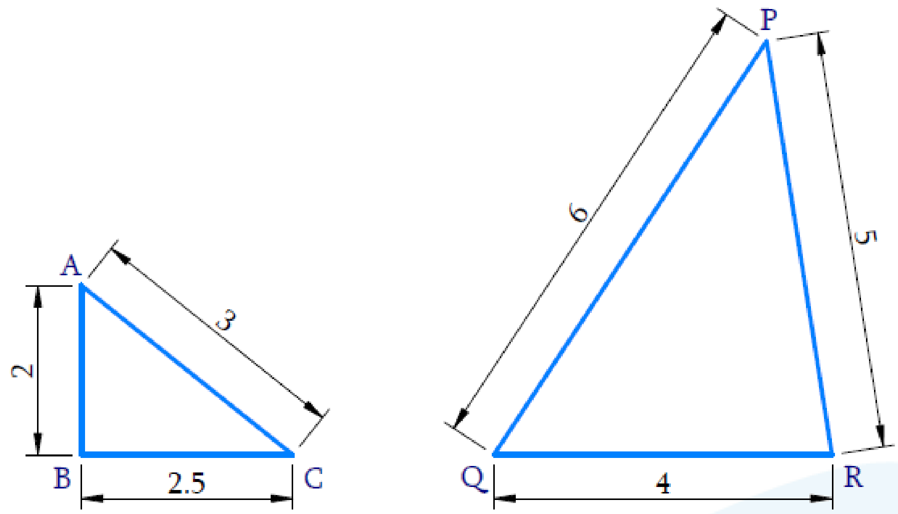
$$\Delta ABC \sim \Delta PQR$$

$\because$ *In* $\Delta\, ABC$ and $\Delta\, PQR \Rightarrow$ All the corresponding angles of the triangles are equal

$\Delta ABC \sim \Delta PQR$

$$\angle A = \angle P = 60^\circ$$
$$\angle B = \angle Q = 80^\circ$$
$$\angle C = \angle R = 40^\circ$$

2)



Reasoning:

Theorem

: If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

This criterion is referred to as the SSS (Side–Side–Side) similarity criterion for two triangles.

Solution:

According to SSS criterion,

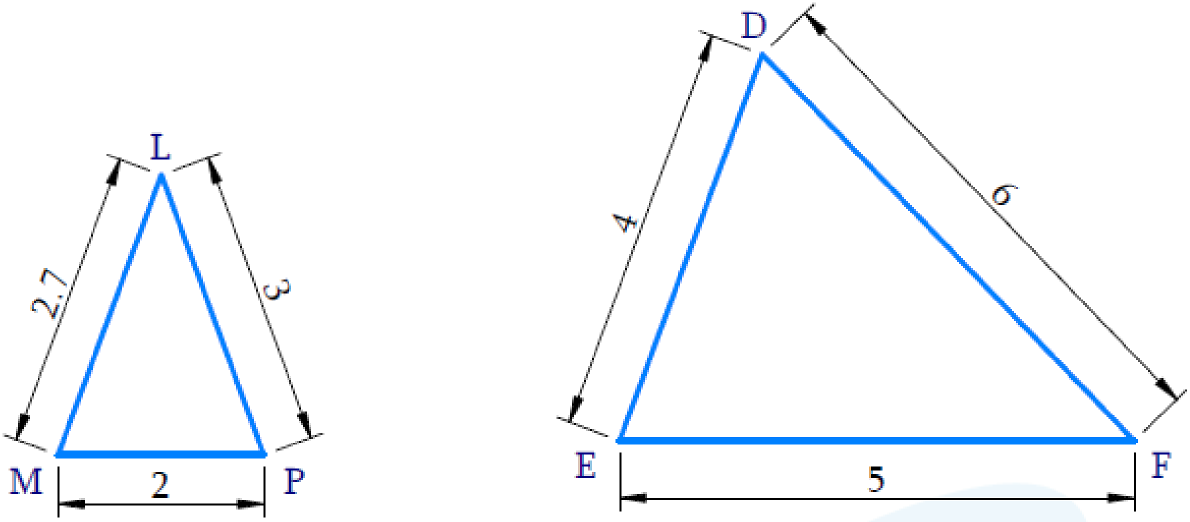
$\Delta ABC \sim \Delta QRP$

$\therefore$ In $\Delta ABC, \Delta QRP$

$$\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}$$
$$\frac{BC}{PR} = \frac{2.5}{5} = \frac{1}{2}$$
$$\frac{AC}{PQ} = \frac{3}{6} = \frac{1}{2}$$
$$\Rightarrow \frac{AB}{QR} = \frac{BC}{PR} = \frac{AC}{PQ} = \frac{1}{2}$$

$\Rightarrow$ All the corresponding sides of two triangle are in same portion.
$\Delta ABC \sim \Delta QPR$

3)



Reasoning:

Theorem 6.4

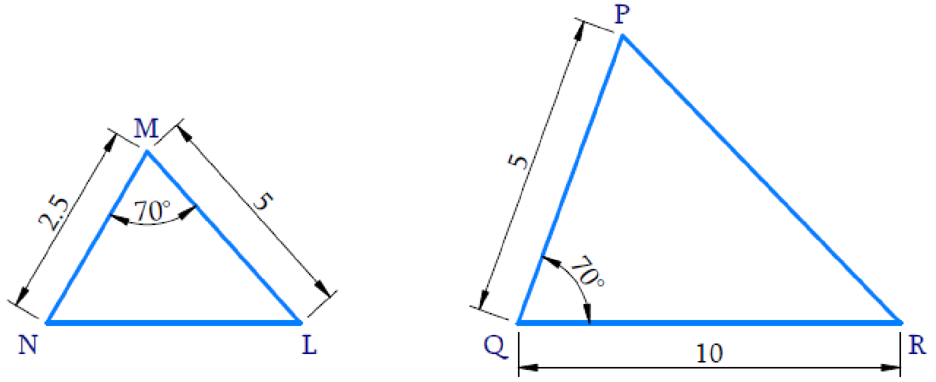
Solution:

$$\Delta LMP \nsim \Delta FED$$
$$\frac{LM}{FE} = \frac{2.7}{5}$$
$$\frac{MP}{ED} = \frac{2}{4} = \frac{1}{2}$$
$$\frac{LP}{FD} = \frac{3}{6} = \frac{1}{2}$$
$$\Rightarrow \frac{LM}{FE} \neq \frac{MP}{ED} \; or \; \frac{LP}{FD}$$

$\Rightarrow$ All the corresponding sides of the two triangles are not in the same proportion.

$$\Rightarrow \Delta LMP \nsim \Delta FED$$

4)



Reasoning:

Theorem 6.5 : If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.
This criterion is referred to as the **SAS (Side–Angle–Side)** similarity criterion for two triangles.

Solution:

Using SAS Criterion,

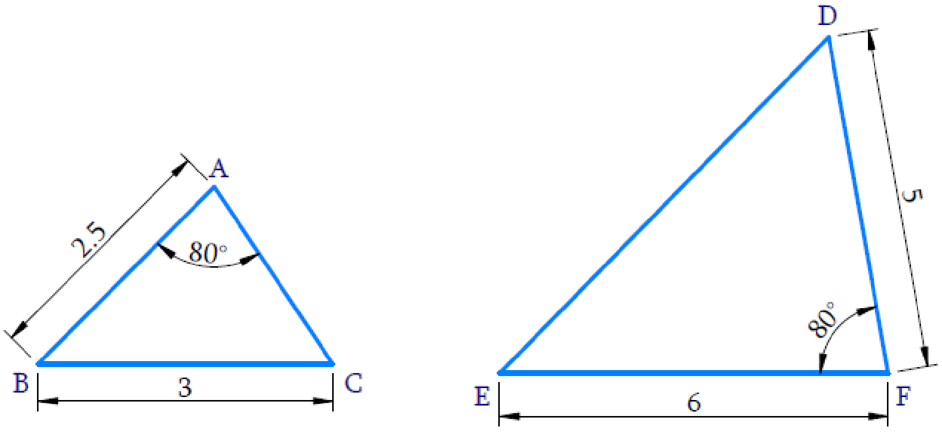
$$\Delta NML \sim \Delta PQR$$

$$In\ \Delta NML,\ \Delta PQR$$

$$\frac{NM}{PQ} = \frac{2.5}{5} = \frac{1}{2}$$
$$\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}$$
$$\Rightarrow \frac{NM}{PQ} = \frac{ML}{QR} = \frac{1}{2}$$
$$\angle M = \angle Q = 70^0$$

$$\Rightarrow \quad \Delta NML \sim \Delta PQR$$

5)



Reasoning:

Theorem 6.5 (Page 134)

Solution:

$\Delta ABC \nsim \Delta DFE$

In $\Delta ABC, \Delta DFE$

$\frac{AB}{DF} = \frac{2.5}{5} = \frac{1}{2}$

$\frac{BC}{EF} = \frac{3}{6} = \frac{1}{2}$

$\frac{AB}{DF} = \frac{BC}{EF} = \frac{1}{2}$

$\angle A = \angle F = 80^\circ$

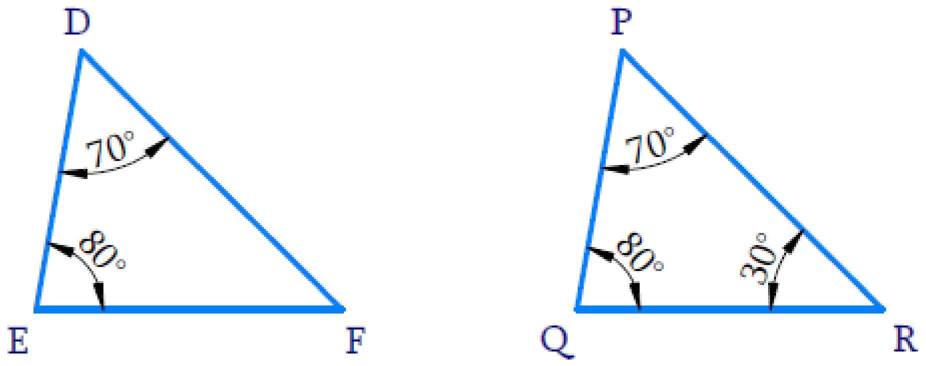
But $\angle B$ must be equal to 80^0

($\because$ The sides AB, BC includes $\angle B$ not $\angle A$)

$\Rightarrow$ SAS criterion is not satisfied

$\Rightarrow \Delta ABC \nsim \Delta DFE$

6)



Reasoning:

Theorem 6.3 : If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.
This criterion is referred to as the AAA (Angle–Angle–Angle) criterion of similarity of two triangles

Solution:

AAA criterion

$$\Delta DEF \sim \Delta PQR$$

In ΔDEF

$$\angle D = 70^0; \angle E = 80^0$$

$\Rightarrow \angle F = 30^0$ [$\because$ Sum of the angles in a Δ is 180^0]

Similarly, In ΔPQR

$$\angle Q = 80^\circ; \quad \angle R = 30^\circ \Rightarrow \angle P = 70^\circ$$

In ΔDEF, ΔPQR

$$\angle D = \angle P = 70^0$$
$$\angle E = \angle Q = 80^0$$
$$\angle F = \angle R = 30^0$$

$\Rightarrow \Delta DEF \sim \Delta PQR$ [AAA Criterion]

Alternate method:

Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This may be referred to as the AA similarity criterion for two triangles..

Solution:

According to AA criterion

$$\Delta DEF \sim \Delta PQR$$

In ΔDEF

$$\angle D = 70^0; \angle E = 80^0$$

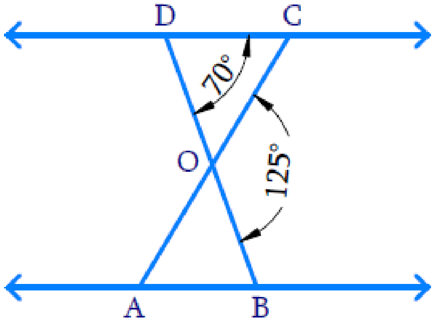
$$\Rightarrow \angle F = 30^0 [\because \text{ Sum of the angles in a } \Delta \text{ is } 180^0 \text{ }]$$

Now ,In $\Delta DEF \sim \Delta PQR$

$$\angle E = \angle Q = 80^0$$
$$\angle F = \angle R = 30^0$$
$$\Rightarrow \Delta DEF \sim \Delta PQR$$

Q2. In Figure 6.35 $\Delta ODC \sim \Delta OBA$, ∠ BOC = 125°and ∠ CDO = 70°. Find ∠DOC, ∠DCO and ∠OAB.

Diagram



Difficulty Level: Medium

Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This may be referred to as the AA similarity criterion for two triangles.

Solution:
In the given figure.

$\angle DOC = 180^\circ - \angle COB$

$[\because \angle DOC \text{ and } \angle COB \text{ from a linear pair}]$

$\angle DOC = 180^\circ - 125^\circ$

$\angle DOC = 55^\circ$

$In\ \Delta ODC$

$\angle DCO = 180^\circ - [\angle DOC + \angle ODC]$

$[\because \text{angle sum property}]$

$\angle DCO = 180^\circ - [55^\circ + 70^\circ]$

$\angle DCO = 55^\circ$

$In\ \Delta ODC, \Delta OBA$

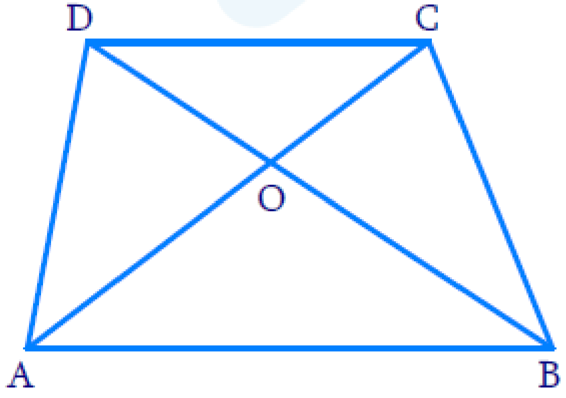
$\Delta ODC \sim \Delta OBA$

$\Rightarrow \angle DCO = \angle OAB\ [\because AA criterion]$

$\angle DCO = 55^\circ$

Q3. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Diagram



Difficulty Level: Medium

Reasoning:

If two angles of one triangle are respectively equal to the two angles of another triangle, then the two triangles are similar. This is referred to as the AA criterion.

Solution

In ΔAOB, ΔCOD

$\angle AOB = \angle COD$ (vertically opposite angles)

$\angle BAO = \angle DCO$ [$\because$ *alternate* interior angles]

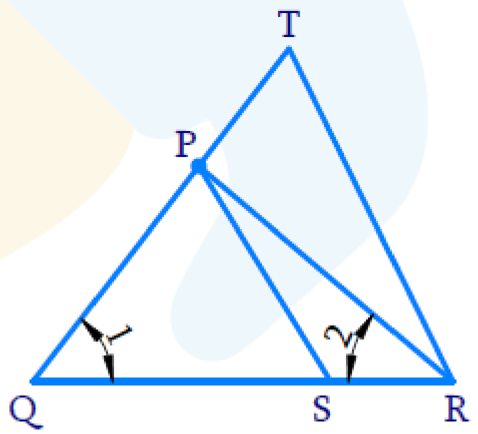
$\Rightarrow \Delta AOB \sim \Delta COD$ [AA criterion]

$$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$$

(**Theorem 6.3 :** If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.
This criterion is referred to as the AAA (Angle–Angle–Angle) criterion of similarity of two triangles.)

Q4. In Figure 6.36 $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.

Diagram



Difficulty Level: Medium

Reasoning:

Theorem 6.5 : If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.
This criterion is referred to as the **SAS (Side–Angle–Side)** similarity criterion for two triangles.

Solution

$$\frac{QR}{QS} = \frac{QT}{PR}$$
$$\angle 1 = \angle 2$$
$$In\ \Delta PQR$$
$$\angle 1 = \angle 2 \Rightarrow PR = PQ$$

($\because$ In a triangle sides opposite to equal angles are equal)

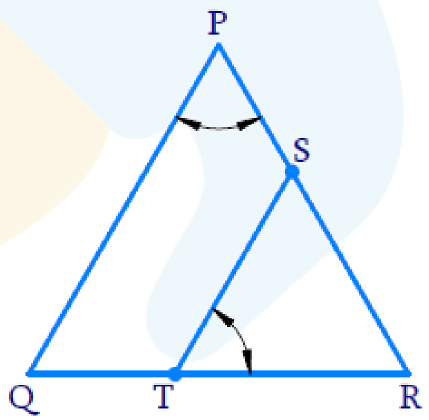
In $\Delta PQS, \Delta TQR$

$$\angle PQS = \angle TQR = \angle 1 \text{[common angle]}$$
$$\frac{QR}{QS} = \frac{QT}{PQ} [\because PR = PQ]$$
$\therefore$ By SAS Criterion,

$$\Rightarrow \Delta PQS \sim \Delta TQR$$

Q5. S and T are points on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.

Diagram



Difficulty Level: Easy

Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This may be referred to as the AA similarity criterion for two triangles.

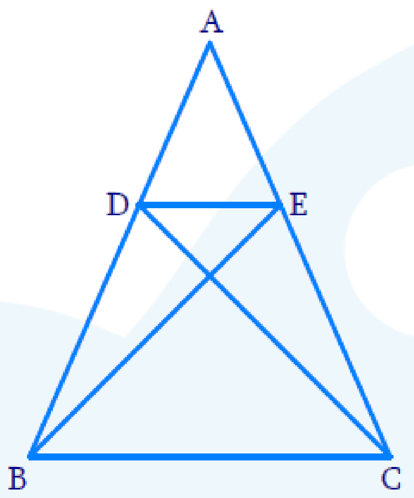
Solution

In $\Delta RPQ, \Delta RTS$

$$\angle RPQ = \angle RTS \quad \text{(given)}$$
$$\angle PRQ = \angle TRS \ (\text{Common angle})$$
$$\Rightarrow \Delta RPQ \sim \Delta RTS \ \text{(AA criterion)}$$

Q6. In Figure 6.37, if Δ ABE ≅ Δ ACD, show that Δ ADE ~ Δ ABC.

Diagram



Difficulty Level: Medium

Reasoning:

Theorem 6.5 : If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.
This criterion is referred to as the **SAS (Side–Angle–Side)** similarity criterion for two triangles.

Solution

In $\Delta ABE, \ \Delta ACD$

$$AE = AD \ (\because \Delta ABE \cong \Delta ACD \ given) ....(1)$$
$$AB = AC \ (\because \Delta ABE \cong \Delta ACD \ given) ....(2)$$

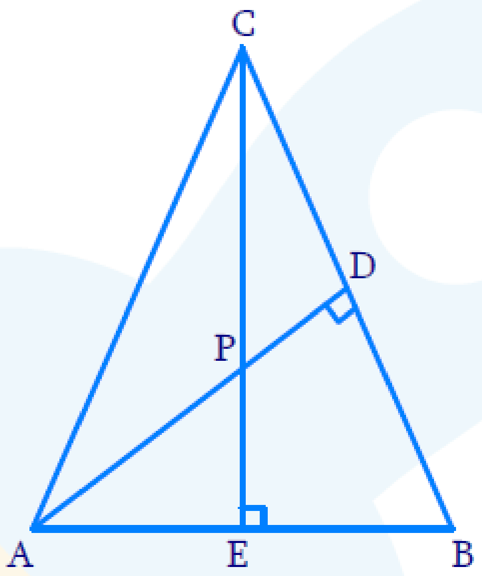
Now Consider $\Delta ADE, \Delta ABC$

$$\frac{AD}{AB} = \frac{AE}{AC} \quad \text{from (1) \& (2)}$$
and $\angle \text{DAF} = \angle \text{BAC}$ (Common angle)
$$\Rightarrow \Delta ADE \sim \Delta ABC \ (\text{SAS criterion})$$

Q7. In Figure 6.38, altitudes AD and CE of ΔABC intersect each other at the point P. Show that:

$(i)\ \Delta AEP \sim \Delta CDP$
$(ii)\ \Delta ABD \sim \Delta CBE$
$(iii)\ \Delta AEP \sim \Delta ADB$
$(iv)\ \Delta PDC \sim \Delta BEC$

Diagram



Difficulty Level: Medium

(i) Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.This may be referred to as the AA similarity criterion for two triangles.

Solution:

$In\ \Delta AEP\ and\ \Delta CDP$

$$\angle AEP = \angle CDP = 90^\circ$$
$$[\because CE \perp AB\ and\ AD \perp BC; altitudes]$$
$$\angle APE = \angle CPD\ (\text{Vertically opposite angles})$$
$$\Rightarrow \Delta AEP \sim \Delta CPD (\text{AA criterion})$$

cuemath

(ii) Reasoning:

AA criterion

Solution

$In\ \Delta ABD,\ \Delta CBE$

$$\angle ADB = \angle CEB = 90^\circ$$
$$\angle ABD = \angle CBE\ (Common\ angle)$$
$$\Rightarrow \Delta ABD \sim \Delta CBE\ (AA\ criterion)$$

(iii) Reasoning:

AA criterion

Solution

$In\ \Delta AEP,\ \Delta ADP$

$$\angle AEP = \angle ADB = 90^\circ$$
$$\angle PAE = \angle BAD\ (\text{Common angle})$$
$$\Rightarrow \Delta\ AEP \sim \Delta\ ADB$$

(iv) Reasoning:

AA criterion

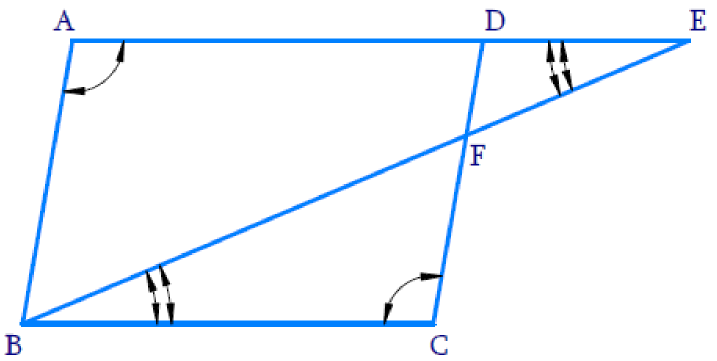
Solution

$In\ \Delta PDC,\ \Delta BEC$

$$\angle PDC = \angle BEC = 90^\circ$$
$$\angle PCD = \angle BCE\ (\text{Common angle})$$
$$\Rightarrow \Delta PDC \sim \Delta BEC$$

Q8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\Delta\ ABE \sim \Delta CFB$.

Diagram



Difficulty Level: Medium

Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This may be referred to as the AA similarity criterion for two triangles.

Solution

In ΔABE, ΔCFB

$\angle BAE = \angle FCB$ (opposite angles of a parallelogram)

$\angle AEB = \angle FBC$ [$\because$ $AE \parallel BC$ and EB is a transversal alternate angle]

$\Rightarrow \Delta ABE \sim \Delta CFE$ (AA criterion)

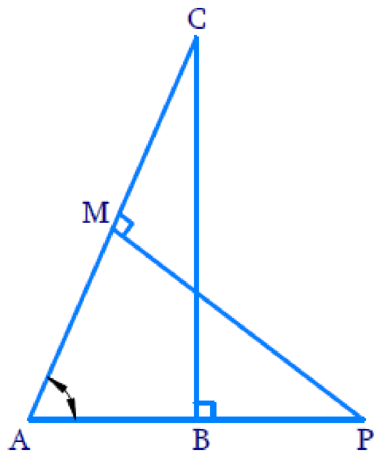
Q9. In Figure 6.39, ABC and AMP are two right triangles, right angled at B and M respectively.

Prove that:

(i) $\Delta ABC \sim \Delta AMP$

(ii) $\dfrac{CA}{PA} = \dfrac{BC}{MP}$

Diagram



Difficulty Level: Medium

(i) Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This may be referred to as the AA similarity criterion for two triangles.

Solution

In ΔABC *and* ΔAMP

$$\angle ABC = \angle AMP = 90^\circ$$
$$\angle BAC = \angle MAP \left(Common\,angle \right)$$
$$\Rightarrow \Delta ABC \sim \Delta AMP$$

(ii) Reasoning:

As we know that the ratio of any two corresponding sides in two equiangular triangles is always the same

Solution

In ΔABC, ΔAMP

$$\frac{CA}{PA} = \frac{BC}{MP} [\because \Delta ABC \sim \Delta AMP]$$

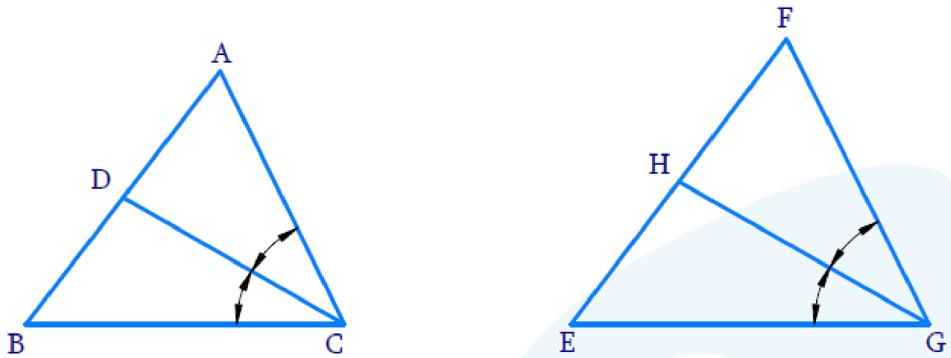
Q10. CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of Δ ABC and Δ EFG respectively. If $\Delta ABC \sim \Delta FEG$, show that:

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) $\Delta DCB \sim \Delta HGE$

(iii) $\Delta DCA \sim \Delta HGF$

Diagram



Difficulty Level: Medium

(i) Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This may be referred to as the AA similarity criterion for two triangles.

Solution

In $\Delta ADC, \Delta FHG$

$$\angle DAC = \angle HFG \left[\because \Delta ADC \sim \Delta FEG\right]$$

$$\angle ACD = \angle FGH \text{ (CD and GH are bisectors } \angle C \text{ and } \angle G \text{ respectively)}$$

$$\left[\angle ACB = \angle FGE \rightarrow \frac{\angle ACB}{2} = \frac{\angle FGE}{2}\right]$$

$$\Rightarrow \Delta ADC \sim \Delta FHG$$

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

[If two triangles are similar, then their corresponding sides are in the same ratio]

(ii) Reasoning:

AA criterion

Solution

In ΔDCB and ΔHGE

$$\angle DBC = \angle HEG [\because \Delta ABC \sim \Delta FEG]$$
$$\angle DCB = \angle HGE \left[\because \frac{\angle ACB}{2} = \frac{\angle FGE}{2}\right]$$
$$\Rightarrow \Delta DCB \sim \Delta EHG (AA \text{ criterion})$$

(iii) Reasoning:

AA criterion

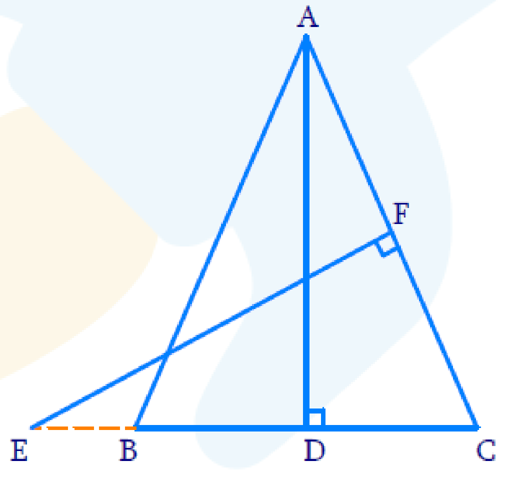
Solution

In ΔDCA, ΔHGF

$$\angle DAC = \angle HFG \; [\because \Delta ABC \sim \Delta FEG]$$
$$\angle ACD = \angle FGH \; [\because \text{CD and GH are bisectors of } \angle ACB \text{ and } \angle FGE]$$
$$\Rightarrow \Delta DCA \sim \Delta HGF \; (\text{AA criterion})$$

Q11. In Figure 6.40, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and $EF \perp AC$, prove that $\Delta ABD \sim \Delta ECF$.

Diagram



Difficulty Level: Medium

Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This may be referred to as the AA similarity criterion for two triangles.

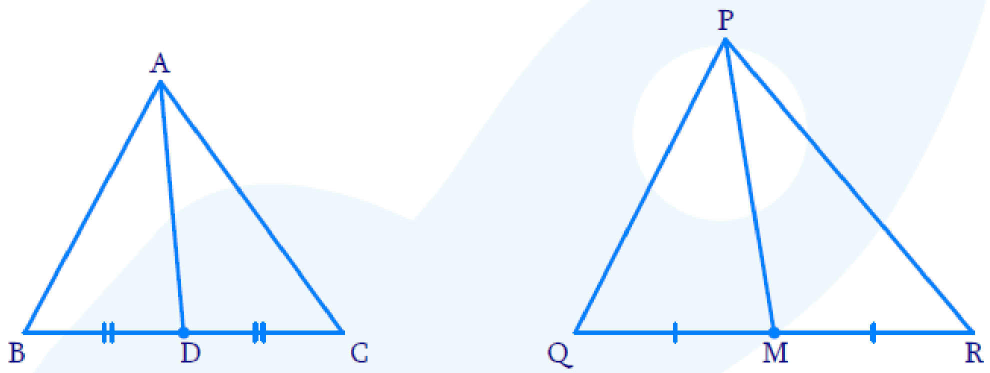
Solution

In ΔABD, ΔECF

$$\angle ADB = \angle EFC = 90^\circ$$
$$[\because AD \perp BC \text{ and } EF \perp AC]$$
$$\angle ABD = \angle ECF$$
$$[\because \text{In } \Delta ABC, AB = AC \Rightarrow \angle ABC = \angle ACB]$$
$$\Rightarrow \Delta ABD \sim \Delta ECF \text{(AA criterion)}$$

Q12. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ , QR and median PM of Δ PQR (see Figure 6.41). Show that Δ ABC ~ Δ PQR.

Diagram



Difficulty Level: Medium

Reasoning:

Theorem 6.5 : If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.
This criterion is referred to as the **SAS (Side–Angle–Side)** similarity criterion for two triangles.

Solution

In ΔABC and ΔPQR

$$\frac{AB}{PQ} = \frac{BC}{QM} = \frac{AD}{PM}$$

Now In $\Delta ABD, \Delta PQM$

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

$$\left[\because \text{AD and PM are median of } \Delta ABC \text{ and } \Delta PQR \Rightarrow \frac{BC}{QR} = \frac{BC/2}{QR/2} = \frac{BD}{QM}\right]$$

$$\Rightarrow \Delta ABD \sim \Delta PQM$$

Now In $\Delta ABC, \Delta PQR$

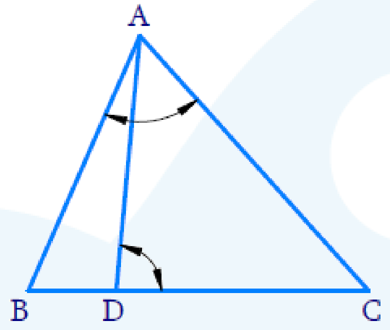
$$\frac{AB}{PQ} = \frac{BC}{QR} \text{(given in the statement)}$$

$$\angle ABC = \angle PQR \; [\because \Delta ABD \sim \Delta PQM]$$

$$\Rightarrow \Delta ABC \sim \Delta PQR \; [SAS\ criteion]$$

Q13. D is a point on the side BC of a triangle ABC such that ∠ADC = ∠ BAC. Show that $CA^2 = CB.CD$.

Diagram



Difficulty Level: Medium

Reasoning:

- If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar. This may be referred to as the AA similarity criterion for two triangles.
- If two triangles are similar then their corresponding sides are in the same proportion.

Solution

In ΔABC *and* ΔDAC

$$\angle BAC = \angle ADC \text{ (Given in the statement)}$$

$$\angle ACB = \angle ACD \text{ (Common angles)}$$

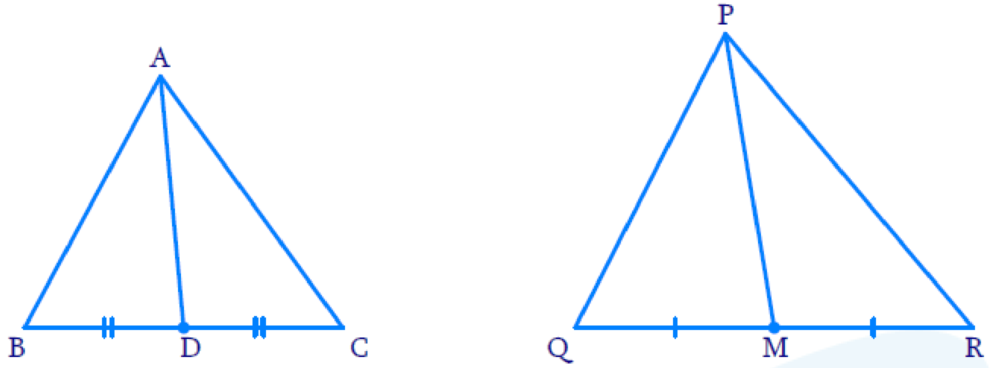
$$\Rightarrow \Delta ABC \sim \Delta DAC \text{ (AA criterion)}$$

$$\Rightarrow \frac{CA}{CD} = \frac{CB}{CA} \text{ [Corresponding sides are in same ratio]}$$

[Sides opposite to equal angles are compared]

$$\Rightarrow CA^2 = CB \cdot CD$$

Q14. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.



Difficulty Level: Medium

Reasoning:

Theorem 6.5 : If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.
This criterion is referred to as the **SAS (Side–Angle–Side)** similarity criterion for two triangles.

Solution

In $\Delta ABC \text{ and } \Delta PQR$

$$\frac{AB}{PQ} = \frac{BC}{QM} = \frac{AD}{PM}$$

Now In $\Delta ABD, \Delta PQM$

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

$$\left[\because \text{AD and PM are median of } \Delta ABC \text{ and } \Delta PQR \Rightarrow \frac{BC}{QR} = \frac{BC/2}{QR/2} = \frac{BD}{QM} \right]$$

$$\Rightarrow \Delta ABD \sim \Delta PQM$$

Now In $\Delta ABC, \Delta PQR$

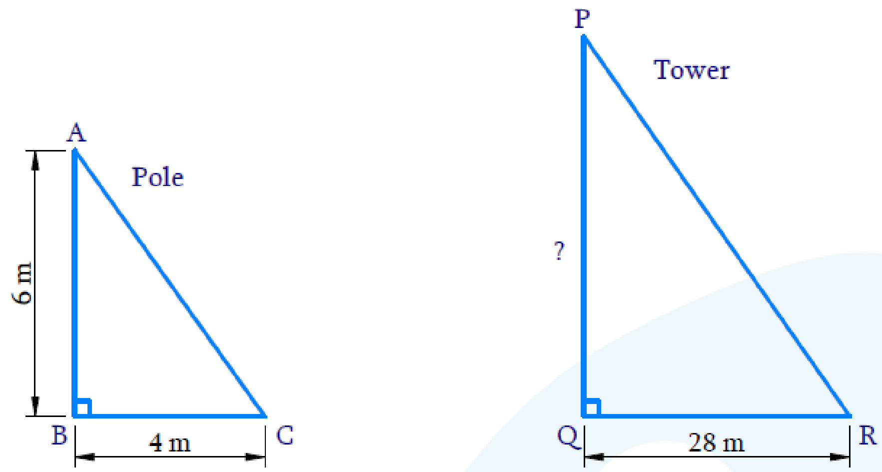
$$\frac{AB}{PQ} = \frac{BC}{QR} (\text{given in the statement})$$

$$\angle ABC = \angle PQR \; [\because \Delta ABD \sim \Delta PQM]$$

$$\Rightarrow \Delta ABC \sim \Delta PQR \; [SAS \; criteion]$$

Q15. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Diagram



Difficulty Level: Medium

Reasoning:

The ratio of any two corresponding sides in two equiangular triangles is always the same.

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This may be referred to as the AA similarity criterion for two triangles.

Solution

AB is the pole = 6m

BC is the shadow of pole = 4m

PQ is the tower =?

QR is the shadow of the tower = 28m

Now in ΔABD and ΔPQR

$\angle ABC = \angle PQR = 90^0$

[∵ The objects and shadow are perpendicular to each other]

$$\angle BAC = \angle QPR$$

[∵ sunrays fall on the pole and tower at the same angle, at the same time]

$$\Rightarrow \Delta ABC \sim \Delta PQR \text{ (AA criterion)}$$

$$\Rightarrow \frac{AB}{BC} = \frac{PQ}{QR}$$

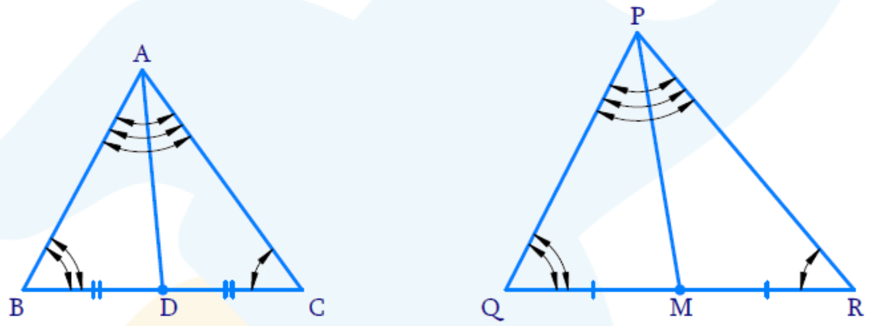
$$\frac{6m}{6m} = \frac{PQ}{28m}$$

$$\Rightarrow PQ = \frac{6 \times 28}{4}$$

$$PQ = 42m$$

Q16. If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AD}{PQ} = \frac{AD}{PM}$.

Diagram



Difficulty Level: Medium

Reasoning:

Theorem 6.5 : If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.
This criterion is referred to as the **SAS (Side–Angle–Side)** similarity criterion for two triangles.

Solution

$$\Delta ABC \sim \Delta PQR$$

$$\Rightarrow \angle ABC = \angle PQR \text{ (corresponding angles)}$$

and $\frac{AB}{PQ} = \frac{BC}{QR}$ (corresponding sides are in the same ratio)

$$\Rightarrow \frac{AB}{PQ} = \frac{BC/2}{QR/2}$$

$$\frac{AB}{PQ} = \frac{BD}{QM}$$ [∵ D and M are mid points of BC and QR]

In $\Delta ABD, \Delta PQM$

$$\angle ABD = \angle PQM \text{ (proved)}$$

$$\frac{AB}{PQ} = \frac{BD}{QM} \text{ (proved)}$$

$$\Rightarrow \Delta ABD \sim \Delta PQM \text{ (SAS criterion)}$$

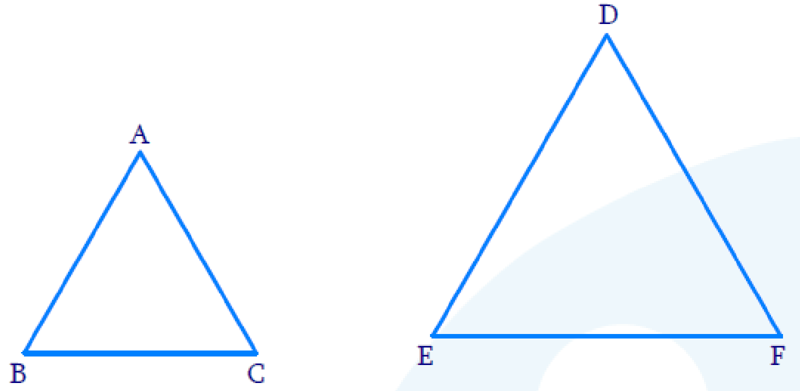
$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \text{ (corresponding sides)}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$$

Exercise 6.4 (Page 143 of Grade 10 NCERT)

Q1. Let $\Delta ABC \sim \Delta DEF$ and their areas be, respectively, $64\,\text{cm}^2$ and $121\,\text{cm}^2$. If EF =15.4 cm, find BC.

Diagram



Difficulty Level: Easy

Reasoning:

Theorem 6.6 : The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Solution

$$\Delta ABC \sim \Delta DEF$$

$$\frac{Area\,of\ \Delta ABC}{Area\,of\ \Delta DEF} = \frac{(BC)^2}{(EF)^2}$$

$$\frac{64cm^2}{121cm^2} = \frac{(BC)^2}{(15.4)^2}$$

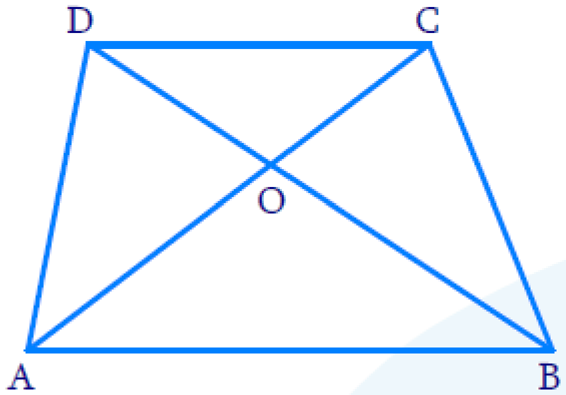
$$(BC)^2 = \frac{(15.4)^2 \times 64}{121}$$

$$BC = \frac{15.4 \times 8}{11}$$

$$BC = 11.2\text{ cm}$$

Q2. Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2 CD, find the ratio of the areas of triangles AOB and COD.

Diagram



Difficulty Level: Medium

Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar. This may be referred to as the AA similarity criterion for two triangles.

Theorem 6.6: The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Solution

In trapezium ABCD, $AB \,||\, CD \text{ and } AB = 2CD$

Diagonals AC, BD intersect at 'O'

In $\Delta AOB, \Delta COD$

$\angle AOB = \angle COD$ (vertically opposite angles)

$\angle ABO = \angle CDO$ [alternate interior angles]

$\Rightarrow \Delta AOB \sim \Delta COD$ (AA criterion)

$$\Rightarrow \frac{Area\,of\,\Delta AOB}{Area\,of\,\Delta COD} = \frac{(AB)^2}{(CD)^2} \quad \text{[theorem 6.6]}$$

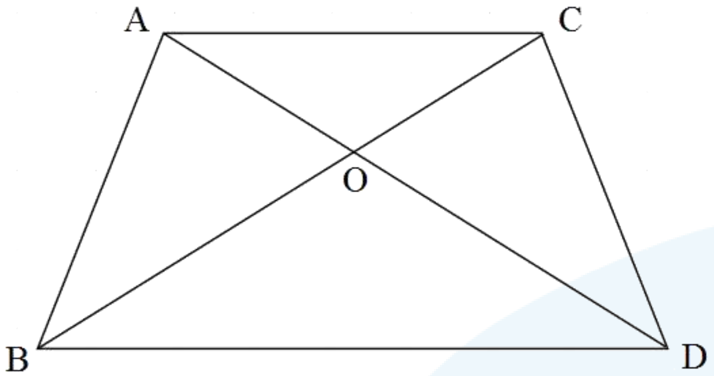
Since AB = 2 CD,

$$= \frac{(2CD)^2}{(CD)^2} = \frac{4CD^2}{CD^2}$$

$\Rightarrow$ Area of ΔAOB : area of ΔCOD = 4:1

Q3. In Fig. 6.44, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{area(ABC)}{area(DBC)}=\frac{AO}{DO}$

Diagram



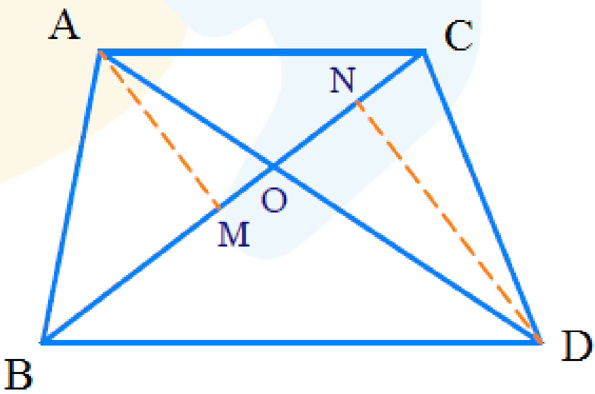
Difficulty Level: Medium

Reasoning:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This may be referred to as the AA similarity criterion for two triangles.

Theorem 6.6 : The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Solution:



In ΔABC

Draw $AM \perp BC$

In ΔDBC

Draw $DN \perp BC$

cuemath

Now in ΔAOM, ΔDON

$$\angle AMO = \angle DNO = 90^\circ$$

$$\angle AOM = \angle DON \left(\text{Vertically opposite angles}\right)$$

$$\Rightarrow \Delta AOM \sim \Delta DON \text{ (AA criterion)}$$

$$\Rightarrow \frac{AM}{DN} = \frac{OM}{ON} = \frac{AO}{DO} \ldots\ldots\ldots\ldots(1)$$

Now,

$$Area\ of\ \Delta ABC = \frac{1}{2} \times base \times height$$

$$= \frac{1}{2} \times BC \times AM$$

$$Area\ of\ \Delta DBC = \frac{1}{2} \times BC \times DN$$

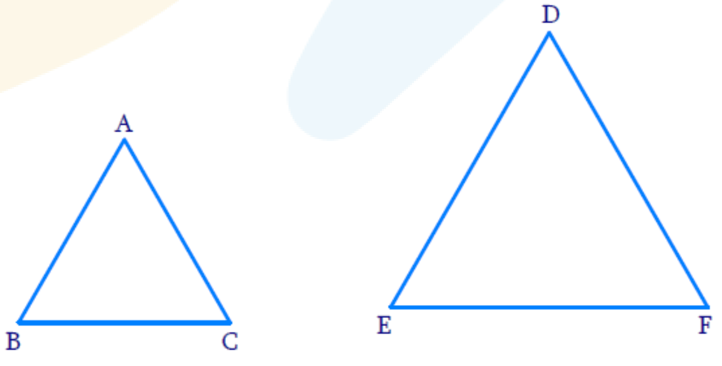
$$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DBC} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times BC \times DN}$$

$$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DBC} = \frac{AM}{DN}$$

$$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DBC} = \frac{AO}{DO} \text{ (from (1))}$$

Q4. If the areas of two similar triangles are equal, prove that they are congruent.

Diagram



Difficulty Level: Medium

Reasoning:

- Two triangular are similar if their corresponding angles are equal and their corresponding sides are in the same ratio.
- **SSS Congruency:** If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.
- **Theorem 6.4:** If in two triangles, sides of one triangle are proportional to(i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similiar.
  This criterion is referred to as the SSS (Side–Side–Side) similarity criterion for two triangles.

Solution:

$$\Delta ABC \sim \Delta DEF$$

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} \text{ (SSS criterion)}$$

But area of ΔABC = area of ΔDEF

$$\Rightarrow \frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF} = 1 \quad ....(1)$$

But $\dfrac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEF} = \dfrac{(AB)^2}{(DE)^2} = \dfrac{(BC)^2}{(EF)^2} = \dfrac{(CA)^2}{(FD)^2}$

From (1)

$$\frac{(AB)^2}{(DE)^2} = \frac{(BC)^2}{(EF)^2} = \frac{(CA)^2}{(FD)^2} = 1$$

$$\Rightarrow \frac{(AB)^2}{(DE)^2} = 1$$

$$\Rightarrow (AB)^2 = (DE)^2$$

$$\Rightarrow AB = DE......................(2)$$

Similarly, BC = EF………………(3)

CA = FD………….......(4)

Now, in $\Delta ABC, \Delta DEF$

AB = DE (form 2)

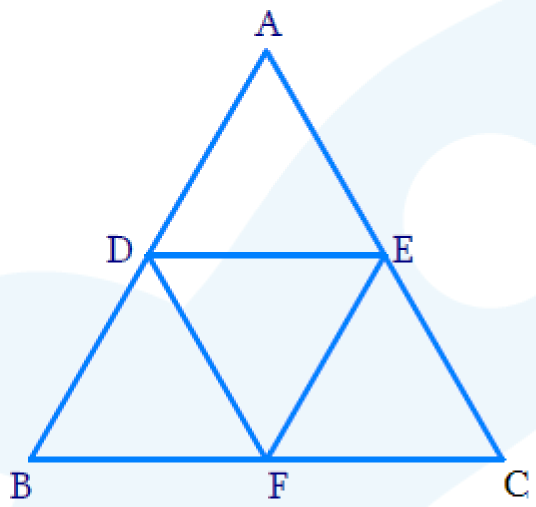
BC = EF (form 3)

CA = FD (form 4)

$\Rightarrow \Delta ABC \cong \Delta DEF$ (SSS congruency)

Q5. D, E and F are respectively the mid-points of sides AB, BC and CA of Δ ABC. Find the ratio of the areas of Δ DEF and Δ ABC.

Diagram



Difficulty Level: Medium

Reasoning:

- As we know that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and half of it – midpoint theorem.
- If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
  This may be referred to as the AA similarity criterion for two triangles.

Solution

In ΔABC D, E are the midpoints of AB, AC

$$\Rightarrow DE \parallel BC \text{ and } DE = \frac{1}{2}BC \ldots\ldots\ldots\ldots..(1)$$

E, F are mid points of AC, BC

$$\Rightarrow EF \parallel AB \text{ and } EF = \frac{1}{2}AB \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

In quadrilateral DBFE,

DE = BF and $DE \parallel BF$ (from 1)

$\Rightarrow$ DBFE is a parallelogram

$\angle B = \angle E$ (opposite angles of a parallelogram are equal)….(3)

Similarly, we can prove that

DFCE is a parallelogram

$\Rightarrow$ $\angle$C = $\angle$D (opposite angles of a parallelogram are equal)……….(4)

Now ,In ΔDEF and ΔABC

$\angle DEF = \angle ABC$ (from 3)

$\angle EDF = \angle ACB$ (from 4)

$\Rightarrow \Delta DEF \sim \Delta ABC$ [AA Criterion]

$\Rightarrow \frac{DE}{BC} = \frac{EF}{AB} = \frac{DF}{AC}$ (The corresponding sides of similar triangles are proportional)

But $\frac{Area\,of\ \Delta DEF}{Area\,of\ \Delta ABC} = \frac{(DE)^2}{(AB)^2} = \frac{(EF)^2}{(AB)^2} = \frac{(DF)^2}{(AC)^2}$

$$\frac{Area\ of\ \Delta DEF}{Area\ of\ \Delta ABC} = \frac{(EF)^2}{(AB)^2}$$

$$= \frac{(\frac{1}{2}AB)^2}{(AB)^2} \text{ [From (2)]}$$

$$= \frac{AB^2}{4AB^2}$$

Area of ΔDEF : Area of ΔABC = 1:4

Alternate method:

Reasoning:

Mid-Point Theorem : The line segment joining the mid-points of two sides of a triangle is parallel to the third side.

Solution:

In ΔABC D and E are midpoints of sides AB and AC

$$\Rightarrow DE \parallel BC \text{ and } DE = \frac{1}{2}BC \text{ ....(1)}$$

Now in quadrilateral DBFE

$$\Rightarrow DE \parallel BC \text{ and } DE = BF \text{ (from 1)}$$

$\Rightarrow$ DBFE is a parallelogram

$\Rightarrow$ Area of ΔDBF = area of ΔDEF ....(2)

($\because$ diagonal DF divides the parallelogram into two triangle of equal area)

Similarly, we can prove

Area of ΔDBF = Area of ΔEFC ....(3)

And area of ΔDEF = Area of ΔADE ....(4)

From (2) (3) and (4)

Area of ΔDBF = Area of ΔDEF = Area of ΔEFC =Area of ΔADE ....(5)

(Things which are equal to the same thing are equal to one another – Euclid's 1st axiom.)

Area of ΔABC = Area of ΔADE + Area of DBF+ Area of ΔEFD +Area of ΔDEF

From (5)

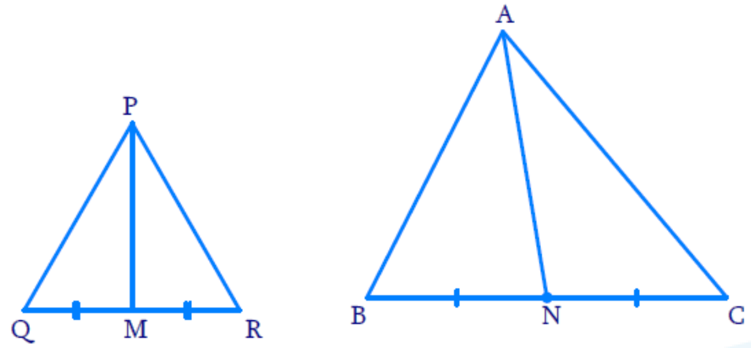
Area of $\Delta ABC = 4 \times$ Area of ΔDEF

$$\frac{Area\ of\ \Delta DEF}{Area\ of\ \Delta ABC} = \frac{1}{4}$$

Area of ΔDEF : Area of ΔABC = 1:4

Q6. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Diagram



Difficulty Level: Medium

Reasoning:

Theorem 6.5 : If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.
This criterion is referred to as the **SAS (Side–Angle–Side)** similarity criterion for two triangles.

Theorem 6.6 : The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Solution:

In ΔPQR ,PM is the median and, in ΔABC AN is the median

$$\Delta PQR \sim \Delta ABC \text{ (given)}$$

$$\angle PQR = \angle ABC ....(1)$$

$$\angle QPR = \angle BAC \ ....(2)$$

$$\angle QRP = \angle BCA \ ....(3)$$

$$\text{and } \frac{PQ}{AB} = \frac{QR}{BC} = \frac{RP}{CA} \ ....(4)$$

($\because$ If two triangles are similar, then their corresponding angles are equal and corresponding sides are in the same ratio)

$$\frac{Area\,of\ \Delta PQR}{Area\,of\ \Delta ABC} = \frac{(PQ)^2}{(AB)^2} = \frac{(QR)^2}{(BC)^2} = \frac{(RP)^2}{(CA)^2} \text{ [from Theorem 6.6].........(5)}$$

Now In ΔPQM and ΔABN

$$\angle PQM = \angle ABN \text{ (from 1)}$$

And $\dfrac{PQ}{AB}=\dfrac{QM}{BN}$

$$\left[\because \frac{PQ}{AB}=\frac{QR}{BC}=\frac{2QM}{2BN};M,\ N\ mid\ points\ of\ QR\ and\ BC\right]$$

$\Rightarrow \Delta PQM \sim \Delta ABN$ [SAS similarity]

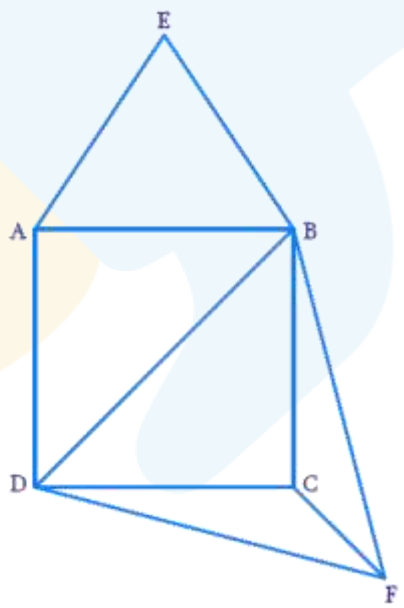
$$\Rightarrow \frac{Area\,of\ \Delta PQM}{Area\,of\ \Delta ABN}=\frac{(PQ)^2}{(AB)^2}=\frac{(QM)^2}{(BN)^2}=\frac{(PM)^2}{(AN)^2}[\because theorem\ 6.6]\ \ldots\ldots..(6)$$

from(5) and (6)

$$\frac{Area\,of\ \Delta PQR}{Area\,of\ \Delta ABC}=\frac{(PM)^2}{(AN)^2}$$

Q7. Prove that the area of an equilateral triangle described on one side of a square is Equal to half the area of the equilateral triangle described on one of its diagonals. Tick the correct answer and justify:

Diagram



Difficulty Level: Medium

Reasoning:

Theorem 6.6 : The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Solution:

ΔABE is described on the side AB of the square ABCD
ΔDBF is described on the diagonal BD of the square ABCD

Since $\Delta ABE \text{ and } \Delta DBF$ are equilateral triangles
[Each angle in an equilateral triangle measures 60°]

$\Delta ABE \sim \Delta DBF$

$$\frac{Area\ of\ \Delta ABE}{Area\ of\ \Delta DBF} = \frac{(AB)^2}{(DB)^2} \quad \text{[Theorem 6.6]}$$

$$= \frac{(AB)^2}{\left(\sqrt{2}AB\right)^2}$$

[$\because$ diagonal of a square is $\sqrt{2} \times$ side]

$$\frac{Area\ of\ \Delta ABE}{Area\ of\ \Delta DBF} = \frac{AB^2}{2AB^2}$$

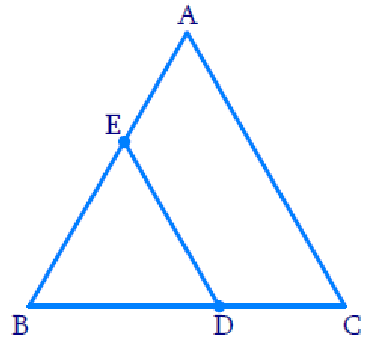
$$\frac{Area\ of\ \Delta ABE}{Area\ of\ \Delta DBF} = \frac{1}{2}$$

$\Rightarrow$ Area of $\Delta ABE = \frac{1}{2}$ Area of ΔDBF

Q8. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is

(A) 2 : 1 (B) 1 : 2 (C) 4 : 1 (D) 1 : 4

Diagram



Difficulty Level: Medium

Reasoning:

Theorem 6.6 : The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Solution: (c)

$\Delta ABC \sim \Delta BDE$ ($\because$ equilateral triangles)

$$\frac{Area\ \Delta ABC}{Area\ \Delta BDE} = \frac{(BC)^2}{(BD)^2}\left[Theorem\ 6.6\right]$$

$$= \frac{(BC)^2}{\left(\frac{BC}{2}\right)^2} \text{ [} \because \text{ D is the midpoint of BC]}$$

$$= \frac{(BC)^2 \times 4}{(BC)^2}$$

$\Rightarrow$ area of ΔABE : area of ΔBDE = 4:1

Q9. Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

(A) 2: 3 (B) 4 : 9 (C) 81 : 16 (D) 16 : 81

Difficulty Level: Easy

Reasoning:

Theorem 6.6 : The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Solution: (d)

We know that,

Ratio of the areas of two similar triangles= square of the ratio of their sides

$$= (4:9)^2$$

$$= 16 : 81$$

Exercise 6.5(Page 150 of Grade 10 NCERT)

Q1. Sides of triangles are given below. Determine which of them are right triangles.
In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm
(ii) 3 cm, 8 cm, 6 cm
(iii) 50 cm, 80 cm, 100 cm
(iv) 13 cm, 12 cm, 5 cm

Difficulty Level: Medium

Reasoning:

Theorem 6.8 : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution

(i) Let us consider,

$$(25)^2 = 625$$

$$\begin{aligned} 7^2 + 24^2 &= 49 + 576 \\ &= 625 \end{aligned}$$

$$\therefore \ (25)^2 = 7^2 + 24^2$$

This is a right triangle as the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Length of hypotenuse = 25cm

(ii)) Let us consider,

$$8^2 = 64$$

$$\begin{aligned} 3^2 + 6^2 &= 9 + 36 \\ &= 45 \end{aligned}$$

$$8^2 \neq 3^2 + 6^2$$

This is not a right triangle as the square of the hypotenuse is not equal to the sum of the squares of the other two sides.

(iv) Let us consider,

$$(100)^2 = 1000$$

$$50^2 + 80^2 = 2500 + 6400$$
$$= 8900$$
$$(100)^2 \neq 50^2 + 80^2$$

This is not a right triangle as the square of the hypotenuse is not equal to the sum of the squares of the other two sides.

(iv) Let us consider,

$$(13)^2 = 169$$

$$12^2 + 5^2 = 144 + 25$$
$$= 169$$

$$\therefore (13)^2 = 12^2 + 5^2$$

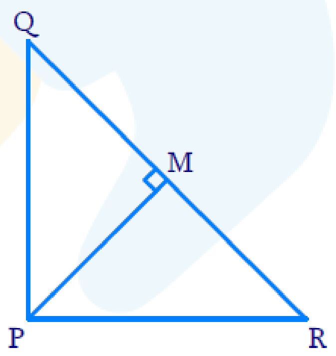
This is a right triangle as the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Length of hypotenuse = 13cm

$\Rightarrow$ (i) and (iv) are right triangle.

Q2. PQR is a triangle right angled at P and M is a point on QR such that PM $\perp$ QR. Show that $(PM)^2$ = QM. MR

Diagram



Difficulty Level: Medium

Reasoning:

Theorem 6.7 : If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This may be referred to as the **AA similarity criterion** for two triangles.

Solution

In $\Delta PQR; \angle QPR = 90^0$ and

$$PM \perp QR$$

In ΔPQR and ΔMQP

$$\angle QPR = \angle QMP = 90^0$$

$$\angle PQR = \angle MQP \text{ (Common Angles)}$$

$$\Rightarrow \Delta PQR \sim \Delta MQP \quad \text{(AA Criterion)} \quad \ldots\ldots\ldots(1)$$

In ΔPQR *and* ΔMPR

$$\angle QMP = \angle PMR = 90^0$$

$$\angle \text{PRQ} = \angle \text{RPM (Common Angle)}$$

$$\Rightarrow \Delta PQR \sim \Delta MPR \text{ (AA Criterion)}\ldots\ldots\ldots(2)$$

From (1) and (2)

$$\Delta MQP \sim \Delta MPR$$

Now In ΔMQP and ΔMPR

$$\angle QMP = \angle PMR = 90^0$$

$$\angle PQM = \angle RPM$$

$$\angle QPM = \angle PRM$$

Comparing corresponding sides
[Comparing sides opposite to equal angles]

$$\frac{PM}{MR} = \frac{QM}{PM}$$

$$\Rightarrow PM^2 = QM.MR$$

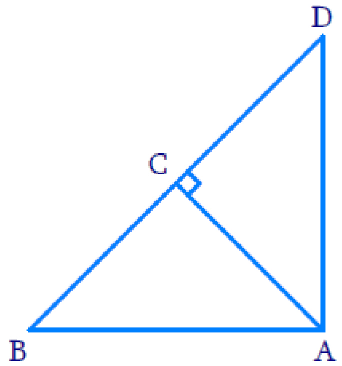
Q3. In Fig. 6.53, ABD is a triangle right angled at A and AC ⊥ BD. Show that

(i) $AB^2 = BC.BD$

(ii) $AC^2 = BC.DC$

(iii) $AD^2 = BD.CD$

Diagram



Difficulty Level: Medium

Reasoning:

Theorem 6.7 : If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This may be referred to as the **AA similarity criterion** for two triangles.

Solution:

i). In $\Delta BAD, \Delta BCA$

$$\angle BAD = \angle BCA = 90^\circ$$
$$\angle ABD = \angle CBA \text{ (common angle)}$$
$$\Rightarrow \Delta BAD \sim \Delta BCA (\text{AA criterion})$$
$$\Rightarrow \frac{AB}{BC} = \frac{BD}{AB} \quad \text{(Corresponding sides of similar triangle)}$$
$$\Rightarrow AB^2 = BC \cdot BD$$

ii). In ΔBCA, ΔACD

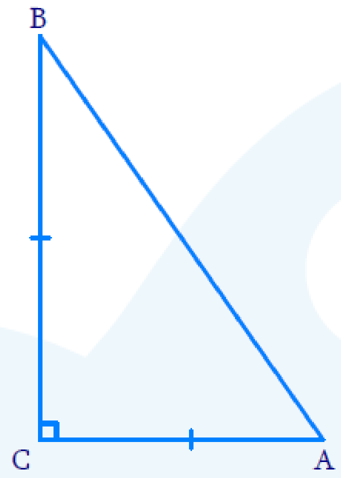
$$\angle BCA = \angle ACD = 90^\circ$$
$$\angle CBA = \angle CAD \text{ (corresponding angle)}$$
$$\Rightarrow \Delta BCA \sim \Delta ACD \ [\text{AA criterion}]$$
$$\Rightarrow \frac{AC}{CD} = \frac{BC}{AC} \text{ [Corresponding sides are similar ratio]}$$
$$\Rightarrow AC^2 = BC \cdot DC$$

iii). In ΔBAD and ΔACD

$$\angle BAD = \angle ACD = 90^\circ$$
$$\angle BDA = \angle ADC \text{ (Common angle)}$$
$$\Rightarrow \Delta BAD \sim \Delta ACD \, [AA\,Criterion]$$
$$\Rightarrow \frac{AD}{CD} = \frac{BD}{AD} \text{[Corresponding sides of similar]}$$
$$AD^2 = BD \cdot CD$$

Q4. ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Diagram



Difficulty Level: Easy

Reasoning:

Theorem 6.8 : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution:

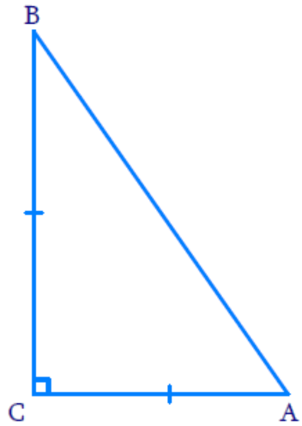
In $\Delta ABC, \ \angle ACB = 90^0$

and $AC = BC$

$$\text{But } AB^2 = AC^2 + BC^2$$
$$= AC^2 + AC^2 [\because AC = BC]$$
$$AB^2 = 2AC^2$$

Q5. ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Diagram



Difficulty Level: Medium

Reasoning:

Theorem 6.9 Converse of the Pythagoras Theorem : In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Solution

In ΔABC

$$AC = BC$$

And $AB^2 = 2AC^2$

$$= AC^2 + AC^2$$

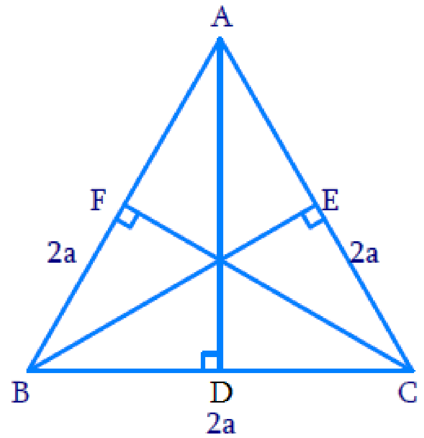
$$AB^2 = AC^2 + BC^2 \; [\because AC = BC]$$

$\Rightarrow \angle ACB = 90^\circ$ Using the Converse of the Pythagoras Theorem

$\Rightarrow \Delta ABC$ is a right triangle

Q6. ABC is an equilateral triangle of side 2a. Find each of its altitudes.

Diagram



Difficulty Level: Medium

Reasoning:

We know that in an equilateral triangle perpendicular drawn from vertex to the opposite side, bisects the side.

Theorem 6.8: In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution

In ΔABC

$AB = BC = CA = 2a$

$AD \perp BC$ [perpendicular drawn from vertex to the opposite side, bisects the side.]

$\Rightarrow BD = CD = a$

In ΔADB, Using Pythagoras Theorem,

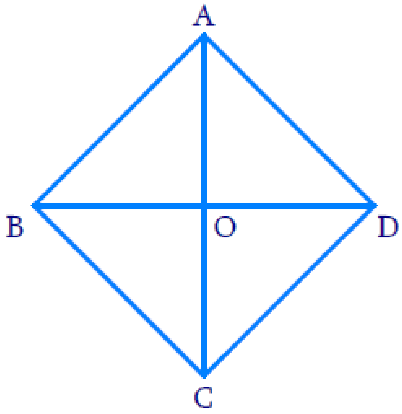
$$\begin{aligned} AB^2 &= AD^2 + BD^2 \\ \Rightarrow AD^2 &= AB^2 - BD^2 \\ &= (2a)^2 - a^2 \\ &= 4a^2 - a^2 \\ AD^2 &= 3a^2 \\ \Rightarrow AD &= \sqrt{3}a \text{ units} \end{aligned}$$

Similarly, we can prove that

$$BE = CF = \sqrt{3}a \text{ units}$$

Q7. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Diagram



Difficulty Level: Medium

Reasoning:

In a rhombus , the diagonals bisect each other perpendicularly.

Theorem 6.8 Pythagoras Theorem : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution:

In rhombus ABCD

$AC \perp BD$ and OA = OC; OB = OD

In ΔAOB

$$\angle AOB = 90^0$$
$$\Rightarrow AB^2 = OA^2 + OB^2 \text{ [Using Pythagoras Theorem]..........(1)}$$

Similarly, we can prove

$$BC^2 = OB^2 + OC^2 \text{...............(2)}$$
$$CD^2 = OC^2 + OD^2 \text{...............(3)}$$
$$AD^2 = OD^2 + OA^2 \text{...............(4)}$$

Adding (1), (2), (3) and (4)

$$AB^2 + BC^2 + CD^2 + AD^2 = OA^2 + OB^2 + OB^2 + OC^2 + OC^2 + OD^2 + OD^2 + OA^2$$

$$= 2OA^2 + 2OB^2 + 2OC^2 + 2OD^2$$

$$= 2[OA^2 + OB^2 + OC^2 + OD^2]$$

$$= 2\left[\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right]$$

$$\left[\because OA = OC = \frac{AC}{2} \text{ and } OB = OD = \frac{BD}{2}\right]$$

$$= 2\left[\frac{AC^2 + BD^2 + AC^2 + BD^2}{4}\right]$$

$$= 2\left[\frac{2AC^2 + 2BD^2}{4}\right]$$

$$= 4\left[\frac{AC^2 + BD^2}{4}\right]$$

$$AB^2 = BC^2 + CD^2 + AD^2$$

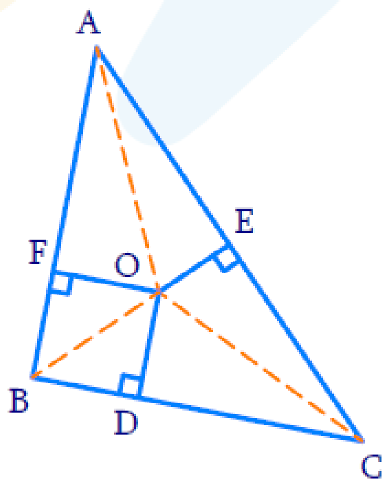
$$= AC^2 + BD^2$$

Q8. In Figure 6.54, O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥AB. Show that

i. $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

ii. $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Diagram



Difficulty Level: Medium

Reasoning:

Theorem 6.8 Pythagoras Theorem : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution:

(i) In ΔABC

$$OD \perp BC, OE \perp AC \text{ and } OF \perp AB$$

OA = OB + OC

In ΔOAF

$$OA^2 = AF^2 + OF^2 [\because \angle OFA = 90^0] \ldots\ldots\ldots\ldots(1)$$

Similarly, In ΔOBD

$$OB^2 = BD^2 + OD^2 [\because \angle ODA = 90^0] \ldots\ldots\ldots\ldots(2)$$

In ΔOCE

$$OC^2 = CE^2 + OE^2 \left[\because \angle OEC = 90^\circ\right] \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Adding (1), (2) and (3)

$$OA^2 + OB^2 + OC^2 = AF^2 + OF^2 + BD^2 + OD^2 + CE^2 + OE^2$$
$$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2 \ldots\ldots\ldots\ldots(4)$$

(ii) From (4)
On Re-grouping,

$$(OA^2 - OE^2) + (OB^2 - OF^2) + (OC^2 - OD^2) = AF^2 + BD^2 + CE^2$$

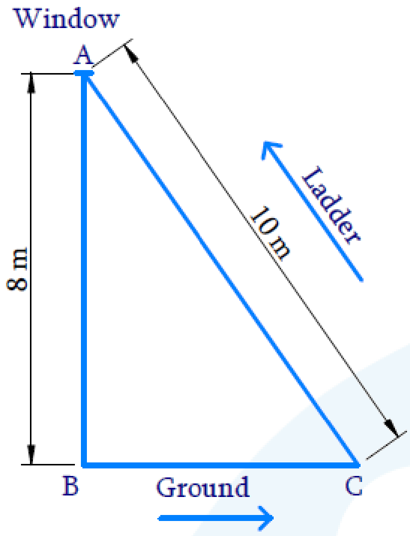
(Rearranging the left side terms)

$$AE^2 + BF^2 + CD^2 = AF^2 + BD^2 + CE^2$$

[$\because$ $\Delta OAE, \Delta OBD$ and ΔOCE are right triangles and by using Pythagoras Theorem it can be mentioned how $OA^2 - OE^2 = AE^2$]

Q9. A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Diagram



Difficulty Level: Easy

Reasoning:

Theorem 6.8 Pythagoras Theorem : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution:

AB is height of the windows from the ground = 8m

AC is the length of the ladder = 10m

BC is the foot of the ladder from the base of ground = ?

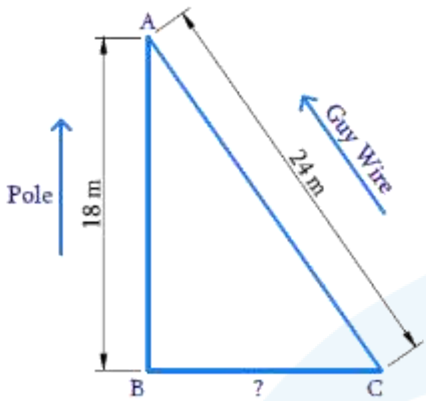
Since ΔABC is right angled triangle ($\angle ABC = 90^0$)

$$\Rightarrow BC^2 = AC^2 - AB^2 \text{ (Pythagoras theorem)}$$
$$= 10^2 - 8^2$$
$$= 100 - 64$$
$$BC^2 = 36$$
$$BC = 6m$$

The distance of the foot of the ladder from the base of the wall =6m

Q10. A guy wire attached to a vertical pole of height 18m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Diagram



Difficulty Level: Easy

Reasoning:

Theorem 6.8 Pythagoras Theorem : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution

AB is the length of the pole = 18m

AC is the length of the guy wire = 24m

BC is the distance of the stake from the pole = ?

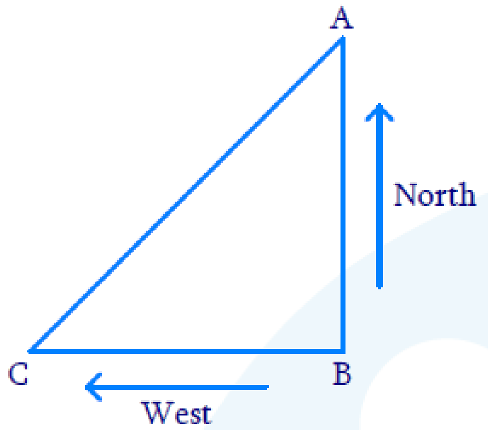
In $\Delta ABC \quad \angle ABC = 90^0$

$$BC^2 = AC^2 - AB^2 \text{ (Pythagoras theorem)}$$
$$= 24^2 - 18^2$$
$$= 576 - 324$$
$$= 252$$
$$BC = 2 \times 3\sqrt{7}$$
$$= 6\sqrt{7}$$

The distance of the stake from the pole $= 6\sqrt{7}m$

Q11. An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Diagram



Difficulty Level: Medium

Reasoning:

We have to find the distance travelled by aeroplanes, we need to use

distance = speed × time

Theorem 6.8 Pythagoras Theorem : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution:

AB is the distance travelled by aeroplanes travelling towards north

$$AB = \text{speed} \times \text{time}$$
$$= 1000\,\text{km/hr} \times 1\frac{1}{2}\,\text{hr}$$
$$= 1000 \times \frac{3}{2}\,\text{km}$$
$$AB = 1500\text{ km}$$

BC is the distance travelled by another aeroplane travelling towards south

$$BC = \text{speed} \times \text{time}$$
$$= 1200 \text{ km/hr} \times 1\frac{1}{2} \text{ hr}$$
$$= 1200 \times \frac{3}{2} \text{hr}$$
$$BC = 1800 \text{ km}$$

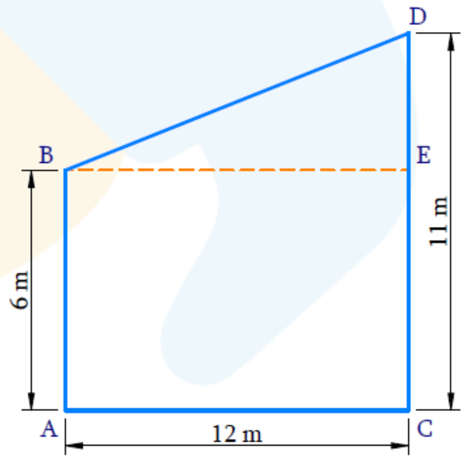
Now, In ΔABC , $\angle ABC = 90^0$

$$AC^2 = AB^2 + BC^2 \left(\text{Pythagoras theorem}\right)$$
$$= (1500)^2 + (1800)^2$$
$$= 2250000 + 3240000$$
$$AC^2 = 5490000$$
$$AC = \sqrt{549000}$$
$$= 300\sqrt{61} \text{ km}$$

The distance between two planes after $1\frac{1}{2}\text{hr} = 300\sqrt{61}$ km

Q12. Two poles of heights 6 m and 11 m stand on plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Diagram



Difficulty Level: Medium

Reasoning:

Theorem 6.8 Pythagoras Theorem : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution:

AB is the height of one pole = 6m

CD is the height of another pole = 11m

AC is the distance between two poles at bottom = 12m

BD is the distance between the tops of the poles =?

Draw $BE \parallel AC$

Now consider ,In ΔBED

$$\angle BED = 90^0$$

BE = AC = 12 m [Opposite sides of a rectangle are equal.]

DE = CD – CE

DE = 11- 6 = 5 cm

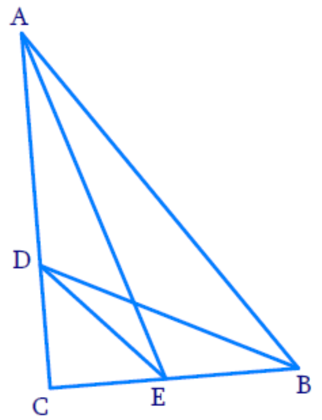
Now

$$BD^2 = BE^2 + DE^2 \left(\text{Pythagoras therorem}\right)$$
$$= 12^2 + 5^2$$
$$= 144 + 25$$
$$BD^2 = 169$$
$$BD = 13\text{m}$$

The distance between the tops of poles =13m

Q13. D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Diagram



Difficulty Level: Medium

Reasoning:

Theorem 6.8 Pythagoras Theorem : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution

In $\Delta ABC, \ \angle ABC = 90^0$

D, E are points on AC and BC

Join AE, DE and BD

In ΔACE,

$AE^2 = AC^2 + CE^2$ (Pythagoras theorem)……….(1)

In ΔDCB

$$BD^2 = CD^2 + BC^2 \text{……………..(2)}$$

Adding (1) and (2)

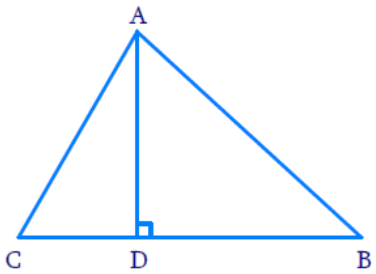
$$\begin{aligned} AE^2 + BD^2 &= AC^2 + CE^2 + CD^2 + BC^2 \\ &= AC^2 + BC^2 + EC^2 + CD^2 \\ &= AB^2 + DE^2 \end{aligned}$$

$$[In\ \Delta ABC, \quad \angle C = 90^0 \Rightarrow AC^2 + BC^2 = AB^2\ and$$
$$In\ \Delta CDE, \angle DCE = 90^0 \Rightarrow CD^2 + CE^2 = DE^2]$$

$$\Rightarrow AE^2 + BD^2 = AB^2 + DE^2$$

Q14. The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3CD(see Fig. 6.55). Prove that $2AB^2 = 2AC^2 + BC^2$.

Diagram



Difficulty Level: Medium

Reasoning:

Theorem 6.8 Pythagoras Theorem : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution:

In ΔABC , $AD \perp BC$

and $BD = 3CD$

In ΔADB

$$AB^2 = AD^2 + BD^2 (\angle ADB = 90^\circ)$$

$$\text{Using } AD^2 = AC^2 - CD^2$$

$$AB^2 = AC^2 + BD^2 - CD^2$$

$$\text{Using } BD = \frac{3BC}{4} \text{ and } CD = \frac{BC}{4}$$

$$= AC^2 + \left(\frac{3BC}{4}\right)^2 - \left(\frac{BC}{4}\right)^2$$

$$AD^2 = \frac{3}{4}BC^2 + \left[\frac{BC}{2} - \frac{BC}{3}\right]^2$$

$$= \frac{3}{4}BC^2 + \left(\frac{BC}{6}\right)^2$$

$$AB^2 = AC^2 + BD^2 - CD^2$$

$$\left[\because \angle ADC = 90^\circ \Rightarrow AC^2 = AD^2 + CD^2\right]$$

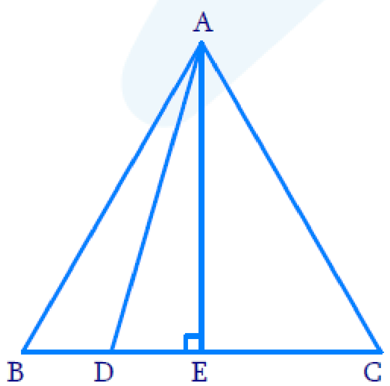
$$\because \ BD + CD = BC$$
$$3CD + CD = BC$$
$$4CD = BC$$
$$CD = \frac{BC}{4}$$
$$and \ \ BD + CD = BC$$
$$\Rightarrow \quad BD + \frac{BD}{3} = BC$$
$$\frac{4BD}{3} = BC$$
$$BD = \frac{3BC}{4}$$

[converting BD and CD in terms of BC]

$$AB^2 = AC^2 + \frac{9BC^2 - BC^2}{16}$$
$$= AC^2 + \frac{8BC^2}{16}$$
$$= AC^2 + \frac{BC^2}{2}$$
$$\Rightarrow 2AB^2 = 2AC^2 + BC^2$$

Q15. In an equilateral triangle ABC, D is a point on side BC such that BD = $\frac{1}{3}BC$ Prove that $9AD^2 = 7AB^2$.

Diagram



Difficulty Level: Medium

Reasoning:

Theorem 6.8 Pythagoras Theorem : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution:

In ΔABC; AB=BC=CA

$$BD = \frac{1}{3}BC$$

and $AD \perp BC$

$$AE = \frac{1}{2}BC$$

[∵ In an equilateral triangle perpendicular drawn from vertex to opposite side bisects the side]

Now In ΔADE

$$AD^2 = AE^2 + DE^2 \text{ (Pythagoras theorem)}$$

$$= (\frac{\sqrt{3}}{2}BC) + (BE - BD)^2$$

[∵ AE is the height of an equilateral triangle which is equal to $\frac{\sqrt{3}}{2}$ side]

Using $BE = \frac{BC}{2}$

$$AD^2 = \frac{3}{4}BC^2 + \left[\frac{BC}{2} - \frac{BC}{3}\right]^2$$

$$= \frac{3}{4}BC^2 + \left(\frac{BC}{6}\right)^2$$

$$AD^2 = \frac{3}{4}BC^2 + \frac{BC^2}{36}$$

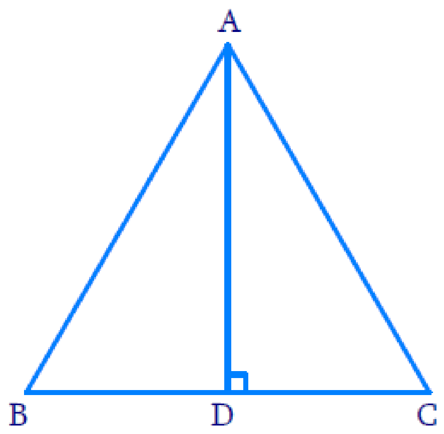
$$= \frac{27BC^2 + BC^2}{36}$$

$$36AD^2 = 28BC^2$$

$$9AD^2 = 7AB^2 \ [\because AB = BC = CA]$$

Q16. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Diagram



We have to prove $3BC^2 = 4AD^2$

Difficulty Level: Medium

Reasoning :

Theorem 6.8 Pythagoras Theorem : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution

In ΔABC

AB = BC = CA

$$AD \perp BC \Rightarrow BD = CD = \frac{BC}{2}$$

Now In ΔADC

$$AC^2 = AD^2 + CD^2$$

$$BC^2 = AD^2 + \left(\frac{BC}{2}\right)^2 \left[AC = BC \text{ and } CD = \frac{BC}{2}\right]$$

$$BC^2 = AD^2 + \frac{BC^2}{4}$$

$$BC^2 - \frac{BC^2}{4} = AD^2$$

$$\frac{3BC^2}{4} = AD^2$$

$$3BC^2 = 4AD^2$$

Q17. Tick the correct answer and justify : In ΔABC, AB = $6\sqrt{3}$ cm , AC = 12 cm and BC = 6 cm. The angle B is

(A) 120° (B) 60° (C) 90° (D) 45°

Difficulty Level: Medium

Reasoning:

Theorem 6.8 Pythagoras Theorem : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
Theorem 6.8

Solution: **(c)**

In ΔABC

$$AB = 6\sqrt{3}\ cm; AC = 12\ cm; BC = 6\ cm$$
$$AB^2 = 108\ cm^2; AC^2 = 144\ cm^2; BC^2 = 36\ cm^2$$

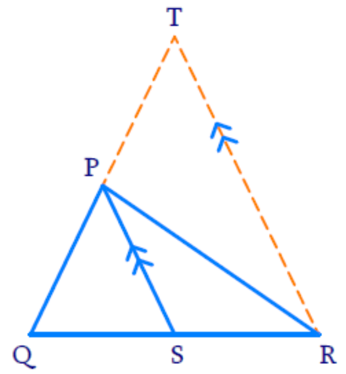
$$AB^2 + BC^2 = (108 + 36)cm^2$$
$$= 144cm^2$$
$$\Rightarrow AC^2 = AB^2 + BC^2$$

Pythagoras theorem is satisfied

$$\Rightarrow \angle ABC = 90^0$$

Exercise 6.6 (Page 152 of Grade 10 NCERT)

Q1. In Fig. 6.50, PS is the bisector of $\angle QPR$ of ΔPQR. Prove that $\frac{QS}{SR} = \frac{PQ}{PR}$



Difficulty Level: Medium

Reasoning:

Theorem 6.1 : If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Solution:

Draw a line parallel to PS, through R, which intersect QP produced at T

Therefore PS || RT

In ΔQPR

$\angle QPS = \angle SPR$ (Since PS is the bisector of $\angle QPR$)...............(i)

$But\ \angle PRT = \angle SPR$ (alternate interior angles)......................(ii)

$\angle QPS = \angle PTR$ (Corresponding angles)...............................(iii)

From (i) , (ii) , (iii)

$\angle PTR = \angle PRT$

PR = PT .... (iv)

(Since in a triangle, sides opposite to the equal angles are equal)

In $\Delta QSP, \Delta QRT$,

$$\Delta QRT \sim \Delta QSP$$

$$\frac{QS}{SR} = \frac{QP}{PT} \quad \text{(Corresponding sides are in same ratio)}$$

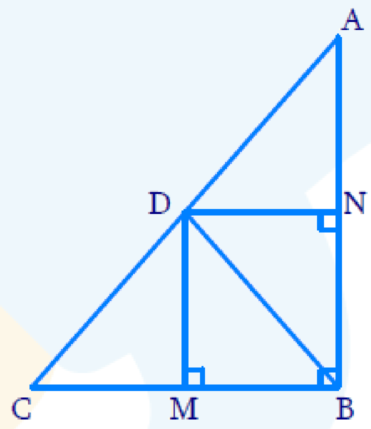
$$\frac{QS}{SR} = \frac{QP}{PR} \quad \text{(From iv)}$$

Q2. In Fig. 6.57, D is a point on hypotenuse AC of ΔABC, such that BD ⊥ AC, DM ⊥ BC and DN ⊥ AB.

Prove that:

(a) $DM^2 = DN.MC$

(b) $DN^2 = DM.AN$



Difficulty Level: Medium

Reasoning:

Theorem 6.1 : If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar. This may be referred to as the **AA similarity criterion** for two triangles.

Solution:

(i) In quadrilateral Δ MBN

$DM \perp BC$ and $DN \perp AB$

DMBN is a rectangle.

DM = BN and DN = MB ...._(i)

In ΔDCM

$$\angle DCM + \angle DMC + \angle CDM = 180^\circ$$
$$\angle DCM + 90^\circ + \angle CDM = 180^\circ$$
$$\angle DCM + \angle CDM = 90^\circ \text{.......................}(ii)$$

But $\angle CDM + \angle BDM = 90^\circ \text{................}(iii)$

Since $BD \perp AC$

From (ii) and (iii)

$$\angle DCM + \angle BDM \text{................(iv)}$$

In ΔBDM

$$\angle DBM + \angle BDM = 90^\circ$$
$$DM \perp BC \text{................(v)}$$

From (iii) and (v)

$$\angle CDM = \angle DBM \text{............}(vi)$$

Now in $\Delta DCM, \Delta DBM$

$\Delta DCM \sim \Delta DBM$ (From 4 and 6 AA criterion)

$\dfrac{BM}{MD} = \dfrac{MD}{MC}$ (Corresponding sides are in same ratio)

$$MD^2 = BM.MC$$
$$MD^2 = DN.MC\left(BM = DN\right)$$

(ii) In ΔBDN

$$\angle BDN + \angle DBN = 90^\circ \left(Since\ DN \perp AB\right) \text{.................}(vii)$$

But $\angle ADN + \angle BDN = 90^\circ \left(Since\ BD \perp AC\right) \text{.................}(viii)$

From (vii) and (viii)

$\angle DBN = \angle ADN$ ________ (ix)

In ΔADN

$$\angle DAN + \angle ADN = 90^\circ \ (Since\ DN \perp AC) \ldots\ldots\ldots (x)$$

But $\angle BDN + \angle ADN = 90^\circ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (xi)$

From (xi) and (x)

$\angle DAN = \angle BDN \ldots\ldots\ldots\ldots\ldots\ldots (xii)$

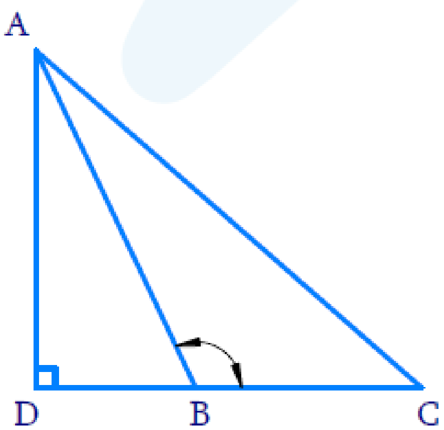
Now in $\Delta BDN, \Delta DAN$,

$\Delta BDN \sim \Delta DAN$ (From ix and xii AA criterion)

$$\frac{BN}{DN} = \frac{DN}{AN} \ (Corresponding\ sides\ are\ in\ same\ ratio)$$
$$DN^2 = BN.AN$$
$$DN^2 = DN.AN \left[BN = DM\right]$$

Q3. In Fig. 6.58, ABC is a triangle in which ∠ABC > 90° and AD ⊥ CB produced.
Prove that:

$$AC^2 = AB^2 + BC^2 + 2BC.BD$$



Difficulty Level: Medium

Reasoning:

Theorem 6.8 Pythagoras Theorem : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution:

In ΔADC

$$\angle ADC = 90^\circ$$

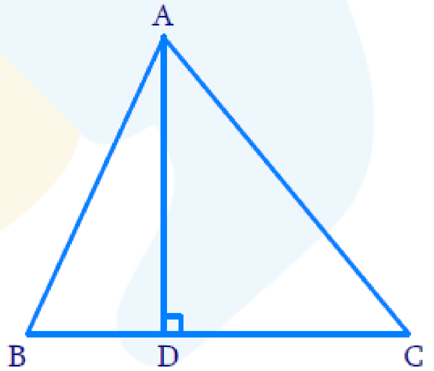
$$\Rightarrow AC^2 = AD^2 + CD^2$$
$$= AD^2 + [BD + BC]^2$$
$$= AD^2 + BD^2 + BC^2 + 2BC \cdot BD$$
$$AC^2 = AB^2 + BC^2 + 2BC \cdot BD$$

($\therefore$ In $\angle ADB$, $AB^2 = AD^2 + BD^2$ by Pythagoras Theorem)

Q4. In Fig. 6.59, ABC is a triangle in which $\angle ABC < 90°$ and AD $\perp$ BC.

Prove that:

$$AC^2 = AB^2 + BC^2 - 2BC \cdot BD$$



Difficulty Level: Medium

Reasoning:

Theorem 6.8 Pythagoras Theorem : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution:

In ΔADC

$$\angle ADC = 90^\circ$$
$$AC^2 = AD^2 + DC^2$$
$$= AD^2 + [BD - BC]^2$$
$$= AD^2 + BD^2 + BC^2 - 2BC.BD$$
$$AC^2 = AB^2 + BC^2 - 2BC.BD$$

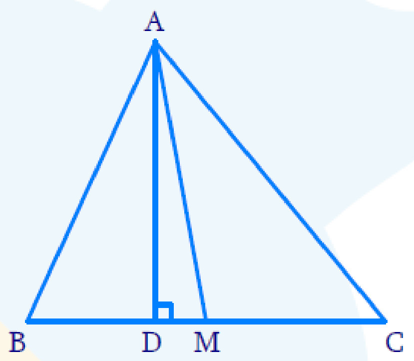
Q5. In Fig. 6.60, AD is a median of a triangle ABC and AM ⊥ BC.

Prove that:

i) $AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$

ii) $AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2$

iii) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$



Difficulty Level: Medium

Reasoning:
Theorem 6.8 Pythagoras Theorem : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution:

(i) In ΔAMC

$$\angle AMC = 90^\circ$$

$$AC^2 = AM^2 + MC^2$$
$$= AM^2 + [MD + CD]^2$$
$$= AM^2 + MD^2 + CD^2 + 2MD.CD$$
$$= AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD.\frac{BC}{2}$$

Since, In $\Delta AMD, \angle AMD = 90^\circ$ and D is the midpoint of BC means BD = CD = $\frac{BC}{2}$

$$AC^2 = AD^2 + \left(\frac{BC}{2}\right)^2 + MD.\text{BC}..............(\text{i})$$

(ii) In ΔAMB Δ AMB

$\angle AMB = 90^\circ$

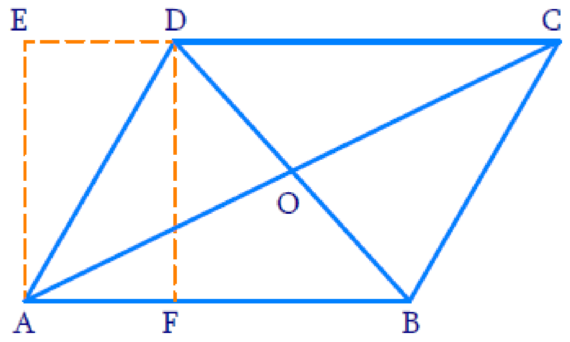
$$AB^2 = AM^2 + BM^2$$
$$= AM^2 + [BD - DM]^2$$
$$= AM^2 + BD^2 + DM^2 - 2BD.DM$$
$$= AM^2 + DM^2 + \left(\frac{BC}{2}\right)^2 - 2\times\frac{BC}{2}\times DM$$

$$AB^2 = AD^2 + \left(\frac{BC}{2}\right)^2 - BC.\text{DM}..............(ii)$$

(iii) Adding (i) and (ii)

$$AC^2 + AB^2 = AD^2 + \left(\frac{BC}{2}\right)^2 + BC.DM + AD^2 + \left(\frac{BC}{2}\right)^2 - BC.DM$$
$$AC^2 + AB^2 = 2AD^2 + 2\left(\frac{BC}{2}\right)^2$$
$$= 2AD^2 + \frac{BC^2}{2}$$

Q6. Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.



Difficulty Level: Medium

Reasoning:

Theorem 6.8 Pythagoras Theorem : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution:

In parallelogram ABCD

AB = CD
AD = BC

Draw $AE \perp CD,\ DF \perp AB$
EA = DF (Perpendiculars drawn between same parallel lines)

In ΔAEC

$$AC^2 = AE^2 + EC^2$$
$$= AE^2 + [ED + DC]^2$$
$$= AE^2 + DE^2 + DC^2 + 2DE.DC$$
$$AC^2 = AD^2 + DC^2 + 2DE \cdot DC \ldots\ldots\ldots\ldots(i)$$

(Since $AD^2 = AE^2 + DE^2$)

In ΔDFB

$$BD^2 = DF^2 + BF^2$$
$$= DF^2 + [AB - AF]^2$$
$$= DF^2 + AB^2 + AF^2 - 2AB.AF$$
$$= AD^2 + AB^2 - 2AB.AF$$
$$BD^2 = AD^2 + AB^2 - 2AB.AF \ldots\ldots\ldots\ldots\ldots(ii)$$

(Since $AD^2 = DF^2 + AF^2$)

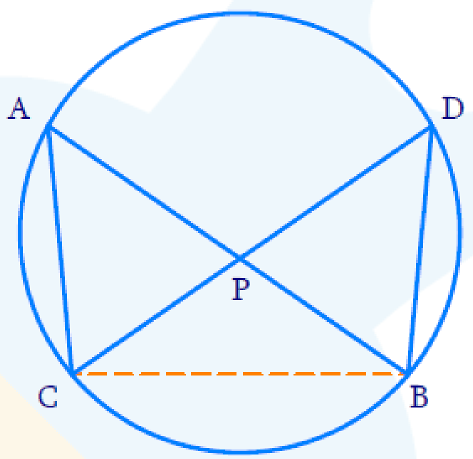
Adding (i) and (ii)

$$AC^2 + BD^2 = AD^2 + DC^2 + 2DE \cdot DC + AD^2 + AB^2 - 2AB.AF$$
$$AC^2 + BD^2 = BC^2 + DC^2 + AD^2 + AB^2 + 2AB.AF - 2AB.F$$

(Since AD = BC and DE = AF , CD = AB)

$$\Rightarrow AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$$

Q7. In Fig. 6.61, two chords AB and CD intersect each other at the point P.
Prove that:
(i) $\Delta APC \sim \Delta DPB$
(ii) AP. PB = CP. DP



Difficulty Level: Medium

Reasoning:

As we know that, two triangles, are similar if :
(i) Their corresponding angles are equal
(ii) Their corresponding sides are in the same ratio

The angles in the same segment of a circle are equal.

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
This may be referred to as the **AA similarity criterion** for two triangles.

Solution:

(i) In ,

$\angle APC = \angle DPB$ (Vertically opposite angles)

$\angle PAC = \angle PDB$ (Angles in the same segment)

$\Rightarrow \Delta APC \sim \Delta DPB$ (A.A criterion)

(ii) In $\Delta APC, \Delta DPB$

$$\frac{AP}{PD} = \frac{PC}{PB} = \frac{AC}{DB} [\Delta APC \sim \Delta DPB]$$

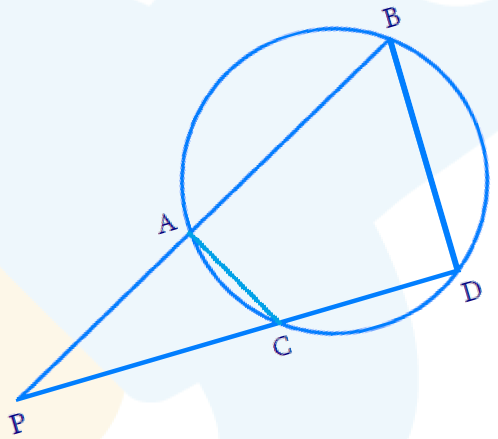
$$\frac{AP}{PD} = \frac{PC}{PB}$$

$\Rightarrow$ AP.PB = PC.PD

Q8. In Fig. 6.62, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that

(i) $\Delta\ PAC \sim \Delta\ PDB$

(ii) PA. PB = PC. PD



Difficulty Level: Medium

Reasoning:

(i) Exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

Solution:

(i) In , $\Delta PAC, \Delta PDB$

$\angle APC = \angle BPD$ (Common angle)

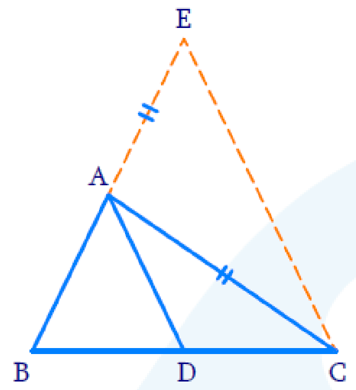
$\angle PAC = \angle PDB$ (Exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.)

$\Rightarrow \Delta PAC \sim \Delta PDB$

(ii) In $\Delta PAC, \Delta PDB$

$$\frac{PA}{PD}=\frac{PC}{PB}=\frac{AC}{BD}$$
$$\frac{PA}{PD}=\frac{PC}{PB}$$
$$PA\cdot PB=PC\cdot PD$$

Q9. In Fig. 6.63, D is a point on side BC of Δ ABC such that $\frac{BD}{CD}=\frac{BA}{CA}$ Prove that AD is the bisector of $\angle BAC$.



Difficulty Level: Medium

Reasoning:

(i) As we know that in an isosceles triangle, the angles opposite to equal sides are equal.

(ii) **Theorem 6.2 Converse of BPT:** If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

Solution:

Extended BA to E such that AE = AC and join CE.

In ΔAEC

AE = AC $\Rightarrow \angle AOE=\angle AEC$ _____ (i)

It is given that

$$\frac{BD}{CD}=\frac{BA}{CA}$$
$$\frac{BD}{CD}=\frac{BA}{AE}\left(\because AC=AE\right)\_\_\_\_\_\_ \quad (ii)$$

In $\Delta ABD, \Delta EBC$

$AD \parallel EC$ (Converse of BPT)

$\Rightarrow \angle BAD = \angle BEC$ (Corresponding angles) _____ (iii)

and $\angle DAC = \angle ACE$ (Alternative Angles) ________ (iv)

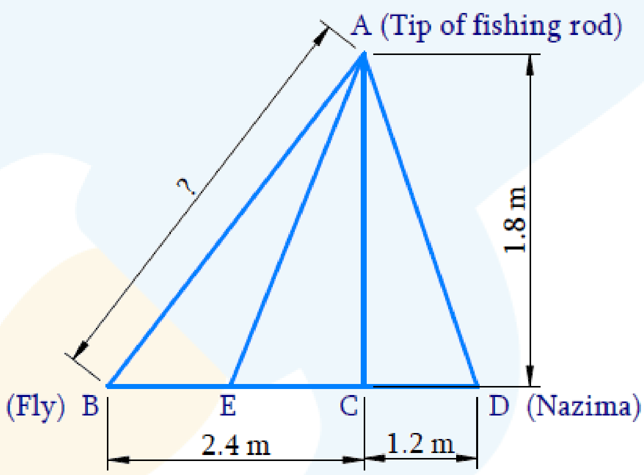
From (i) , (iii) and (iv)

$\angle BAD = \angle DAC$

$\Rightarrow AD$ is the bisector of $\angle BAC$

.

Q10. Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Fig. 6.64)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?



Difficulty Level: Medium

Reasoning:

Theorem 6.8 Pythagoras Theorem : In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Solution:

To find AB and ED
BD = 3.6 m , BC = 2.4 m , CD = 1.2 m

AC = 1.8 cm

In ΔACB

$$AB^2 = AC^2 + BC^2$$
$$= (1.8)^2 + (2.4)^2$$
$$= 3.24 + 5.76$$
$$AB^2 = 9.00$$

Length of the string out AB= 3cm
Let the fly at E after 12 seconds
String pulled in 12 seconds = 12 × 5
= 60 cm
=0.6 m
AE = 3m – 0.6 m
= 2.4 m

Now In ΔACE

$$CE^2 = AE^2 - AC^2$$
$$= (2.4)^2 - (1.8)^2$$
$$CE^2 = 5.76 - 3.24$$
$$= 2.52$$
$$CE = 1.587m$$

DE = CE + CD
= 1.587 + 1.2
= 2.787
DE = 2.79 m

Horizontal distance of the fly after 12 seconds = 2.79 m