Arithmetic Integers

Review Exercise Questions

Level-1

A – SINGLE CHOICE

Q1. The HCF of $2^2\times3^3\times5^2$, $2^3\times3^2\times5^2\times7$ and $2^4\times3^4\times5\times7^2\times11$ is:

A. $2^2\times3^2\times5$

B. $2^2\times3^2\times5\times7\times11$

C. $2^4\times3^4\times5^5$

D. $2^4\times3^4\times5^4\times7\times11$

E. None of these

Q2. The HCF of $2^3\times3^2\times5\times7^4$, $2^2\times3^5\times5^2\times7^3$ and $2^3\times5^3\times7^2$ is:

A. 980

B. 908

C. 890

D. 809

E. None of these

Q3. The HCF of 108, 288 and 360 is:

A. 63

B. 33

C. 36

D. 66

E. None of these

Q4. The LCM of $2^2\times3^3\times5\times7^2$, $2^3\times3^2\times5^2\times7^4$ and $2\times3\times5^3\times7\times11$ is:

A. $2^2\times3^3\times5^3\times7^4\times11$

B. $2^3\times3^2\times5^3\times7^4\times11$

C. $2^3\times3^3\times5^3\times7^4\times11$

D. $2^3\times3^3\times5^2\times7^4\times11$

E. None of these

Q5. The LCM of 72, 108 and 2100 is

A. 38700

B. 37800

C. 78300

D. 73800

E. None of these

Q6. The LCM of the numbers

$$2^{2m} \times 3^{4n} \times 5^{p}$$

$$2^{m} \times 3^{2n} \times 5^{3p}$$

$$2^{3m} \times 5^{3p} \times 7^{2m+n}$$

is

A. $2^{2m} \times 3^{6n} \times 5^{3p} \times 7^{2m+n}$

B. $2^{3m} \times 3^{4n} \times 5^{3p} \times 7^{2m+n}$

C. $2^{2m} \times 3^{2n} \times 5^{p} \times 7^{2m+n}$

D. $2^{3m} \times 3^{4n} \times 5^{p} \times 7^{2m+n}$

Here, m, n and p are natural numbers.

Q7. Which of the following has the most number of distinct prime factors?

A. 99

B. 101

C. 176

D. 182

Q8. If the sum of two numbers is 55 and the HCF and LCM of these numbers are 5 and 120 respectively, then the sum of the reciprocals of the numbers is equal to:

A. $\frac{55}{601}$

B. $\frac{601}{55}$

C. $\frac{11}{120}$

D. $\frac{120}{11}$

E. None of these

Q9. The sum of the LCM and HCF of two numbers is 1260. If their LCM is 900 more than their HCF, then the product of the two numbers is

A. 203400

B. 194400

C. 198400

D. 20540

Q10. If p, q are two consecutive natural numbers, then $\text{HCF}(p, q)$ is

A. q

B. p

C. 1

D. $p \times q$

E. None of these

Q11. If p, q are two prime numbers, then LCM (p, q) is

A. 1
B. p
C. q
D. $p \times q$
E. None of these

Q12. The HCF of two numbers is 48, and the HCF of two other numbers is 36. Then, the HCF of all four numbers is

A. 4
B. 6
C. 12
D. 8
E. None of these

Q13. It is known that

$$HCF \text{ of several fractions} = \frac{HCF \text{ of their numerators}}{LCM \text{ of their denominators}}$$

The HCF of the fractions

$$\frac{8}{21}, \frac{12}{35}, \frac{32}{7} \quad \text{is}$$

A. $\frac{4}{105}$
B. $\frac{192}{7}$
C. $\frac{4}{7}$
D. $\frac{5}{109}$
E. None of these

Q14. It is known that

$$LCM \text{ of several fractions} = \frac{LCM \text{ of their numerators}}{HCF \text{ of their denominators}}$$

The LCM of the fractions

$$\frac{5}{16}, \frac{15}{24}, \frac{25}{8} \quad \text{is}$$

A. $\frac{5}{48}$
B. $\frac{5}{8}$
C. $\frac{75}{48}$
D. $\frac{75}{8}$
E. None of these

Q15. Which of the following is a pair of co-prime numbers?

A. $(14, 35)$

B. $(18, 25)$

C. $(31, 93)$

D. $(32, 62)$

E. None of these

Q16. Which of the following will leave the largest remainder upon division by 7?

A. -1
B. -2
C. -3
D. -4

B – MULTIPLE CHOICE

Q17. The product of any four consecutive integers will always be a multiple of which of the following?

A. 6
B. 8
C. 12
D. 24

Which of the following are incorrect statements?

A. If n is divisible by 4 and 3, then it is divisible by 12.
B. If n is divisible by 4 and 6, then it is divisible by 24.
C. Even if n is not a multiple of 3, it is possible that $2n$ is a multiple of 3.
D. If n is even, then $3n$ is a multiple of 6.
E. If $7n$ is divisible by 3, then n must be divisible by 3.

C – INTEGER ANSWERS

Q18. If the prime factorization of 3600 is of the form

$$2^p \times 3^2 \times 5^q \times 7^l$$

then the value of $p+q+l$ is __________

Q19. The smallest composite number that has four different prime factors is __________

Q20. If the HCF of two numbers $2^3 \times 3^2 \times 7^p$ and $5 \times 3^q \times 7^2$ is 21, then the value of $3p+5q$ is __________

Q21. If the HCF of two numbers $2^p \times 3^4 \times 5$ and $2^3 \times 3^q \times 7$ is 108, then the value of $p+q$ is __________

Q22. If the HCF of two numbers $2^4 \times 3^p \times 5$ and $2^q \times 3^2 \times 43$ is 24, then the sum of the two numbers is __________

Q23. If the LCM of two numbers $2^p \times 3^2 \times 17$ and $2 \times 3^q \times r$ is 44676, then the magnitude of the difference of the numbers is __________

Q24. The product of two numbers is 4107. If the HCF of these numbers is 37, then the greater number is __________

Q25. Three number are in the ratio of 3: 4 : 5 and their LCM is 2400. Their HCF is __________

Q26. The product of two numbers is 2028 and their HCF is 13. The number of such pairs is __________

Q27. The HCF of two numbers is 11 and their LCM is 7700. If one of the numbers is 275, then the other is __________

Q28. The ratio of two numbers is 3:4 and their HCF is 4. Their LCM is __________

Q29. The LCM of two numbers is 48. The numbers are in the ratio $2:3$. Then, the sum of the numbers is __________

Q30. Consider the following statements:

I. The LCM of x and 18 is 36.

II. The HCF of x and 18 is 2.

The value of x is _________

Q31. If p is the largest number which divides 248 and 1032 leaving a remainder of 8 in each case, then the value of p is __________

Q32. If p is the largest number which divides 546 and 764 leaving remainders of 6 and 8 respectively, then the value of p is __________

Q33. Two tanks have a capacity of 504 and 735 litres of milk respectively. The maximum capacity of a container which can measure the milk of either tank an exact number of times (in litres) is __________

Q34. The sum of the powers of 2 and 3 in the prime factorization of 8640 is __________

Q35. The HCF of 1120 and 1512 is __________

Q36. The LCM of 225 and 280 is __________

Q37. The LCM of 60, 80 and 110 is __________

Q38. The LCM of 120, 150 and 180 is __________

Q39. The minimum value of n for 10^n to be a multiple of 256 is __________

Q40. The minimum value of n for 21^n to be a multiple of 343 is __________

Q41. The minimum value of n for 6^n to be a multiple of 729 is __________

Q42. The smallest positive integer which leaves a remainder of 3 upon division by 4 and 10 upon division by 11 is__________.

Q43. The smallest positive integer greater than 10 which leaves a remainder of 10 upon division by both 13 and 17 is __________

Q44. A number n leaves a remainder of 1 upon division by 3. The remainder obtained when n^2 is divided by 3 will be __________

Q45. The square of an odd integer is divided by 4. The sum of all the possible remainders is __________

Q46. A number n leaves a remainder of 1 upon division by 5. If n^2 is divided by 10, the sum of all the possible remainders is __________

Q47. The HCF of 8448 and 5082 is calculated using Euclid's Division Algorithm. The dividend in the last step of the algorithm is __________

Q48. The HCF of 6699 and 5655 is calculated using Euclid's Division Algorithm. The dividend in the last step of the algorithm is __________

Q49. When -13 is divided by 11, the remainder is __________

Q50. The HCF of 7854 and 4746 is calculated using Euclid's division algorithm. In how many steps does the algorithm terminate? __________

D – MISCELLANEOUS

Q51. For a positive integer n, let $S(n)$ denote the sum of the positive divisors of n, and let $G(n)$ be the greatest divisor of n. If

$$H(n) = \frac{G(n)}{S(n)}$$

then which is larger: $H(100)$ or $H(101)$?

Q52. There will exist a unique pair of numbers with any given values of their HCF and LCM. Is this true or false?

Q53. Every odd number can be written in the form $4k+3$, where k is some integer. Is this true or false?

Q54. Prove that in three consecutive integers, one must be a multiple of 3.

Q55. Prove that in any three odd consecutive integers, one must be an odd multiple of 3.

Q56. In any three consecutive even integers, it is not necessary for one of them to be a multiple of 6. Is this true or false?

Q57. Prove that in any four consecutive even integers, there will be exactly one multiple of 8.

Q58. In any four consecutive odd integers, one must be a multiple of 5. Is this true or false?

Q59. Any integer can be written either as $3k$ or $3k+1$. Is this true or false?

Q60. The product of any three consecutive integers will always be a multiple of 6. Is this true or false?

Q61. Prove that if n is an odd integer, then n^2-1 must be divisible by 8.

Q62. Show that every odd multiple of 3 can be written as $4k+1$ or $4k-1$ for some appropriate value of k.

Q63. (a) Prove that for a positive integer m, we will have $(ma,mb)=m(a,b)$.

(b) If d is a positive common factor of a and b, prove that $\left(\frac{a}{d},\frac{b}{d}\right)=\frac{(a,b)}{d}$.

Q64. Prove that if a and b are relatively prime to n, then so is ab.

Q65. Find integer values of a and b such that $30a-41b=1$.

Q66. It is given that m and n are two integers such that

$$(m,4)=2,\ (n,4)=2$$

Find the value of $(m+n,4)$.

Q67. p is a prime number. Find all possible integer solutions of the equation $m^2-n^2=p$.

Q68. What is the number of zeroes at the end of 1000!?

Q69. Consider the number $n=3\times 2^9$. Which numbers from the set {2, 5, 6, 8, 9} are factors of n?

Q70. Prove that if a number has an odd number of divisors, then it is a perfect square.

Q71. Prove that there are infinitely many prime numbers.