Chapter - 13: Surface Area and Volumes

Exercise 13.1 (Page 213 of Grade 9 NCERT Textbook)

Q1. Plastic box 1.5 m long, 1.25 m wide and 65 cm deep, is to be made. It is opened at the top. Ignoring the thickness of the plastic sheet, determine:

i. The area of the sheet required for making the box.
ii. The cost of sheet for it, if a sheet measuring $1m^2$ costs Rs. 20.

Difficulty Level: Medium

Reasoning:

(i) The outer surface of a cuboid is made up of six rectangles called the faces whose areas can be found by multiplying length and breadth for each of them separately and then adding six areas together. Since the box is opened at the top, it has only 5 surfaces.

(ii) The cost of the sheet to create the box will be equal to total surface area of the box multiplied by cost per meter squared of the sheet.

Known:
Area of the sheet required for making the box.

Unknown:
The length, breadth and depth of the plastic bag to be made.

Solution:

Length(l) = 1.5 m
Breadth(b) = 1.25 m
Height(h) = 65 cm = 0.65m [100 cm = 1 m]
$\therefore$ All the units should be the same
The area of the open box $= lb + 2(bh + hl)$

$$= (1.5 \times 1.25) + 2\,[(1.25 \times .65) + (.65 \times 1.5)]$$
$$= 1.875 + 2(.8125 + .975)$$
$$= 1.875 + 2(1.7875)$$
$$= 1.875 + 3.575$$
$$= 5.45 m^2$$

Answer:
The area of the sheet required for making the open box in $= 5.45\ m^2$

Unknown: Cost of the sheet if $1m^2$ costs Rs. 20

cuemath

Solution:

$1m^2$ costs $= Rs.\ 20$

$5.45m^2$ costs $= Rs.\ 20 \times 5.45$

$= Rs.\ 109$

Answer:

Cost of the sheet is Rs. 109

Q2. The length, breadth and height of a room are 5 m, 4 m, and 3 m respectively. Find the cost of white washing the walls of the room and ceiling at the rate of Rs 7.50 per m^2 ?

Reasoning:

A cuboid is enclosed by six rectangular regions called the faces of the cuboid. Surface area of the cuboid of length l, breadth b and height h is $2(lb + bh + hl)$ The four walls and ceiling are to be white washed. So it is also having 5 faces only. For finding the cost of white washing, the surface area should be found and then should be multiplied by the cost for whitewashing per meter square.

Known:

The length, breadth and height of the cuboid.

Unknown:

Cost of white washing the ceiling and the walls Rs 7.50 per m^2.

Solution:

Surface area of 5 faces $= lb + 2(bh + hl)$

$length = 5m$

$breadth = 4m$

$height = 3\ m$

Area of the 4 walls and ceiling $= 5 \times 4 + 2(5 + 4)3$

$= 20 + 54$

$= 74m^2$

Cost of white washing the walls of the room and the room and the ceiling

$= 74 \times 7.50$

$= Rs.\ 555$

Answer:

Cost of white washing the walls of the room and the ceiling = Rs. 555

Q3. The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs. 10 per m^2 is Rs. 15000, find the height of the hall. [Hint: Area of the four walls = Lateral surface area.]

Difficulty Level: Medium

Reasoning:

Lateral surface area of the cuboid is only the area of 4 walls of the cuboid. Lateral surface area of cuboid $= 2(l+b)h$. So, the ratio between the total cost for painting and cost per m^2 will give the total lateral surface area painted.

Known:

i. Perimeter of the hall which is $2(l+b) = 250\ m$
ii. Cost of painting at the rate of Rs.10 per m^2 is Rs. 15000.

Unknown:

Height of the hall.

Solution:

The ratio between the total cost for painting and cost per m^2 will give the total lateral surface area painted.

Area of four walls = $\dfrac{15000}{10} = 1500$

Perimeter = $[2\,(l+b)]$ = 250 m

$$2(l+b)h = 1500$$
$$(250 \times h) = 1500$$
$$h = \frac{1500}{250}$$
$$= 6\ m$$

Answer:

The height of the hall is 6 m.

Q4. The paint in a certain container is sufficient to paint an area equal to 9.375 m^2 . How many bricks of dimensions 22.5 cm × 10 cm × 7.5 cm can be painted out of this container?

Reasoning:

Brick is nothing but a cuboid having six faces. Surface area of the brick is the sum of the 6 faces. So, the total number of bricks that can be painted will be given by the ratio of total area of container divided by the surface area of per brick.

Known:

Dimensions of brick. The area can be painted with the paint.

Unknown:

Number of bricks can be painted.

Solution:

For a brick:
l = 22.5 cm
b = 10 cm
h = 7.5 cm

The total surface area of the brick:

$$= 2(lb + bh + hl)$$
$$= 2(22.5 \times 10 + 10 \times 7.5 \times 7.5 \times 22.5)$$
$$= 2(225 + 75 + 168.75)$$
$$= 2(468.75) = 937.5\ cm^2$$

Since the area given in m^2. So, the area of the brick has to be changed to m^2.
Surface area of the brick $= 937.5\ cm^2$

$$= \frac{937.5}{100 \times 100} m^2$$
$$= 0.09375\ m^2$$

Number of bricks can be painted $= \dfrac{9.375}{0.09375} = 100$

Answer:
Number of bricks that can be painted from the paint in the container = 100.

Q5. A cubical box has each edge 10 cm and another cuboidal box is 12.5 cm long, 10 cm wide and 8 cm high.

i. Which box has the greater lateral surface area and by how much?
ii. Which box has the smaller total surface area and by how much?

Reasoning:

A cube is cuboid whose length, breadth and height and equal. A cuboid has six faces and the total surface area is the sum of the surface area of the 6 faces.

Known:

i. The length of the cube.
ii. The length, breadth height of the cuboid.

Unknown:

Greater lateral surface area and by how much?

Solution:

Lateral surface area of the cube and cuboid is the sum of the area of the four faces.
For cube:
Edge of the cube is 10 cm.
Lateral surface area of the cube $= 4\ (edge)^2$

$$= 4 \times 10^2$$
$$= 400\ cm^2$$

For cuboid:
$length(l) = 12.5\ cm$
$breadth(b) = 10\ cm$
$height(h) = 8\ cm$
Lateral surface area of the cuboid $= 2(l+b)\ h$
Lateral surface area of the cuboid $= 2\ (12.5+10) \times 8$

$$= 2(12.5+10) \times 8$$
$$= 2 \times 22.5 \times 8$$
$$= 16 \times 22.5$$
$$= 360\ cm^2$$

Answer:
Cubical box has the greater lateral surface area than the cuboidal box by $40\ cm^2$.
$(400 - 360 = 40\ cm^2)$.

ii. Smaller total surface area

Known:

i. The length of the cube.
ii. The length, breadth height of the cuboid.

Unknown:

Smaller total surface area and by how much?

Solution:
Total surface area of the cube $= 6\ (edge)^2$

$$= 6 \times 10^2$$
$$= 600\ cm^2$$

Total surface area of the cuboid $= 2(lb + bh + hl)$

$length(l) = 12.5\ cm$
$breadth(b) = 10\ cm$
$height(h) = 8cm$

Total surface area:

$$= 2\,[(12.5 \times 10 + 10 \times 8 + 8 \times 12.5)]$$
$$= 2\,[125 + 80 + 100]$$
$$= 610\ cm^2$$

Answer:

Cubical box has the smaller total surface area than the cuboidal box by $(610 - 600) = 10cm^2$

Q6. A small indoor greenhouse (herbarium) is made entirely of glass panes (including base) held together with tape. It is 30 cm long, 25 cm wide and 25 cm high.

i. What is the area of the glass?
ii. How much of tape is needed for all the 12 edges?

Reasoning:

A cuboid is enclosed by six rectangle regions called faces and it has 12 edges. The total surface area is the sum of the areas of the six faces which is nothing but the area of the glass.

i. Area of the glass?

Known:

Measurements of herbarium.

Unknown:

The area of the glass.

Solution:

The area of the herbarium is the total surface area of the cuboid.

$length(l) = 30cm$

$breadth = 25cm$

$height = 25cm$

Total surface area of the cuboid

$= 2(lb + bh + hl)$

$= 2[25 \times 30 + 25 \times 25 + 30 \times 25]$

$= 2[750 + 625 + 750]$

$= 4250\ cm^2$

Answer:

Area of the glass $= 4250\ cm^2$.

ii. How much of tape is needed for all the 12 edges?

Unknown:

Tape needed for all the 12 edge.

Solution:

Total length of the tape needed for 12 edges

$= 4(l + b + h)$

$= 4(30 + 25 + 25)$

$= 320\ cm$

Answer:

Tape required to cover all the 12 edges is 320 cm.

Q7. Shanti Sweets Stall was placing an order for making cardboard boxes for packing their sweets. Two sizes of boxes were required. The bigger of dimensions 25 cm × 20 cm × 5 cm and the smaller of dimensions 15 cm × 12 cm × 5 cm. For all the overlaps, 5% of the total surface area is required extra. If the cost of the cardboard is Rs 4 for 1000 cm^2, find the cost of cardboard required for supplying 250 boxes of each kind.

Difficulty Level: Hard

Reasoning:

A cuboid has six faces and the total surface area is sum of the area of the six faces. So, the cost for supplying 250 boxes of each kind will be the summation of surface area of boxes multiplied by cost per cm square.

Known:

i. Dimensions of the smaller and bigger boxes.
ii. Cost of the card board.

Unknown:

Cost of the card board required for 250 boxes of each kind.

Solution:

For bigger box:

$length\ (l) = 25\ cm$
$breadth\ (b) = 20\ cm$
$height\ (h) = 5\ cm$
Total surface area $= 2\,(lb + bh + hl)$
$= 2\,[25 \times 20 + 25 \times 20 + 5 \times 25]$
$= 2\,[500 + 100 + 125]$
$= 1450\ cm^2$

Card board required for all the overlaps is 5% of their total surface area.
$\therefore \frac{5}{100} \times 1450 = 72.5\ cm$

Net cardboard required for bigger box $= 1450 + 72.5$ $= 1522.5\ cm^2$

Card board required for 250 such boxes $= 1522.5 \times 250$ $= 380625\ cm^2$
Cost for $1000\ m^2 = Rs.\ 4$
$\therefore$ Cost for $380625\ cm^2 = \frac{4}{1000} \times 380625$
$= Rs.\ 1522.5$

For smaller box:

$length\ (l) = 15\ cm$
$breadth\ (b) = 12\ cm$
$height\ (h) = 5\ cm$

Total surface area $= 2(lb + bh + hl)$
$= 2[50 \times 12 + 12 \times 5 + 5 \times 15]$
$= 2[180 + 60 + 75]$
$= 630\ cm^2$

Card board required for all the overlaps is 5% of their total surface area.
$\therefore \frac{5}{100} \times 630 = 31.5\ cm^2$

Net surface area of the smaller box $= 630 + 31.5$ $= 661.5\ cm^2$
Card board required for 250 such boxes $= 661.5 \times 250$ $= 165375\ cm^2$

Cost of the card board is Rs. 4 for $1000\ cm^2$

For $165375\ cm^2$ cost is $= \frac{4}{1000} \times 16537$

$= Rs.\ 661.50$

Answer:

Cost of card board required for supplying 250 boxes of each kind

= 1522.50 + 661.50

= Rs. 2184

Q8. Parveen wanted to make a temporary shelter for her car, by making a box-like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m, with base dimensions 4 m × 3 m?

Reasoning:

The surface area of the cuboid is the sum of the surface area of the faces. Since Parveen wanted to make a cuboid with 5 outer faces covered with tarpaulin that covers all the four side and the top.

Known:

Measurements of the shelter needed is given.

Unknown:

Area of the tarpaulin required to make the shelter.

Solution:

$length(l) = 4\ m$
$breadth(b) = 3\ m$
$height(h) = 2.5\ m$

Total surface area with five faces $= lb + 2(bh + hl)$

$= 4 \times 3 + 2(3 \times 2.5 + 2.5 \times 4)$

$= 12 + 2[7.5 + 10]$

$= 47\ cm^2$

Answer:

Hence $47\ cm^2$ of tarpaulin will be required.

Exercise 13.2 (Page 216 of Grade 9 NCERT Textbook)

Assume $\pi = \frac{22}{7}$ unless stated otherwise

Q1. The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the base of the cylinder.

Reasoning:

Curved surface area of cylinder $= 2\pi rh$

Known:

Curved surface area of the cylinder and its height.

Unknown:

Diameter of the base of the cylinder.

Solution:

Curved surface area of a cylinder $= 2\pi rh = 88 cm^2 \left(given\right)$

Height of the cylinder =14 cm

$$2 \times \frac{22}{7} \times r \times 14 = 88$$

$$r = \frac{88 \times 7}{2 \times 22 \times 14} = 1cm$$

$$2 \times radius = diameter$$

$$2 \times 1 = 2cm$$

Answer:

The diameter of the base of the cylinder = 2 cm

Q2. It is required to make a closed cylindrical tank of height 1 m and base diameter 140 cm from a metal sheet. How many square metres of the sheet are required for the same?

Reasoning:

The curved surface area of a right circular cylinder of base r and height h is 2πrh and its total surface area is $2\pi r(r+h)$

Known:

Height and diameter of the cylindrical tank.

Unknown:
Square metres of the sheet required for making the cylindrical tank.

Solution:

Height of the tank (h) = 1 m = 100 cm
Diameter = 2r = 140 cm

$r = \frac{140}{2} = 70\ cm$

Total surface area of the closed cylindrical tank $= 2\pi r(r+h)$

$$= 2 \times \frac{22}{7} \times 70(70+100)$$
$$= 74800\ cm^2$$

Since 100 cm = 1 m

$$100^2 cm^2 = 1\ m^2$$
$$74800\ cm^2 = \frac{748000}{100 \times 100}$$
$$= 7.48\ m^2$$

Answer:
$7.48\,m^2$ of the sheet is required for making the cylindrical tank.

Q3. A metal pipe is 77 cm long. The inner diameter of a cross section is 4 cm, the outer diameter being 4.4 cm (see Fig. 13.11). Find its

i. Inner curved surface area
ii. Outer curved surface area
iii. Total surface area

Reasoning:

The curved surface area of a right circular cylinder of base radius r and height is 2πrh. And its total surface area is $2\pi r(r+h)$.

Known:

Length of the metal pipe, the inner and outer diameter of the cross sector of the metal pipe.

i. Inner curved surface area

Solution:

$height(h) = 77\ cm$

$diameter = 2r = 4\ cm$

$r = 2\ cm$

Inner curved surface area $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 2 \times 77$

$= 968\ cm^2$

ii. Outer curved surface area

Solution:

$height(h) = 77\ cm$

$diameter = 2R = 4.4\ cm$

$R = 2.2\ cm$

Inner curved surface area $= 2\pi Rh$

$= 2 \times \frac{22}{7} \times 2.2 \times 77$

$= 1064.8\ cm^2$

iii. Total surface area

Solution:

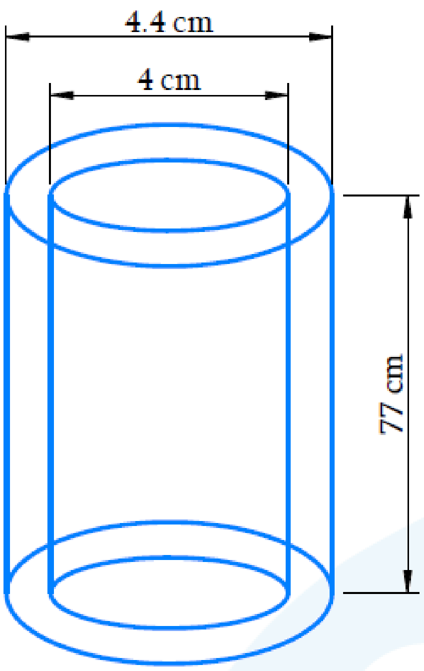
Total surface area of pipe = Curved Surface Area of inner surface + Curved Surface Area of outer surface + Area of both the circular ends of the pipe

Total surface area $= 2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$

$= 1064.8 + 968 + 2 \times \frac{22}{7} \times [(2.2)^2 - (2)^2]$

$= 1064.8 + 968 + 5.28$

$= 2038.08\ cm^2$



Answer:

Inner curved surface area $= 968\ cm^2$

Outer curved surface area $= 1064.8\ cm^2$

Total surface area $= 2038.08\ cm^2$

Q4. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground in m^2.

Reasoning:

The roller is cylindrical in shape and hence it is considered as a right circular cylinder.
The curved surface area of a right circular cylinder of base radius r and height h is $2\pi rh$

Known:

Radius and height of the cylinder.

Unknown:

Area of the playground in m^2.

Solution:

Curved surface area of the cylinder = $2\pi rh$

$$diameter = 2r = 84\ cm$$
$$r = 42\ cm$$
$$h = 120\ cm$$

Area of the playground leveled in taking 1 complete revolution = The curved surface area of the cylinder having r = 42cm and h = 120cm.

$$= 2\pi rh$$
$$= 2 \times \frac{22}{7} \times 42 \times 120$$
$$= 31680\ cm^2$$

Area of the playground = Area levelled by the cylinder in 500 revolutions.

$$= 500 \times 31680$$
$$= 15840000\ cm^2$$

$$Since\ 100\ cm = 1\ m$$
$$100^2\ cm^2 = 1\ m^2$$

$$= \frac{15840000}{100 \times 100}$$
$$= 1584\ m^2$$

Answer:
Area of the play ground $= 1584\ m^2$

Q5. A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of Rs 12.50 per m^2.

Reasoning:

The curved surface area of a right circular cylinder of base radius r and height h is $2\pi rh$. So, the cost of painting curved surface area will be the product of curved surface are and cost of painting per meter square.

Known:

Height and diameter of the pillar.

Unknown:

Cost of painting the curved surface of the pillar.

Solution:

Diameter = 2r = 50 cm
r = 25 cm = 0.25 m
Height = h = 3.5 m
Curved surface area = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 0.25 \times 3.5$$

$$= 5.5\ m^2$$

Cost of painting the curved surface area per m^2 = Rs 12.50
Cost of painting 5.5 m^2 $= 12.50 \times 5.5 = Rs\ 68.75$

Answer:

Cost of painting the curved surface of the pillar is Rs. 68.75.

Q6. Curved surface area of a right circular cylinder is 4.4 m^2. If the radius of the base of the cylinder is 0.7 m, find its height.

Reasoning:

The curved surface area of a right circular cylinder of the base radius r and height h is $2\pi rh$.

Known:

Curve surface area and radius of the base.

Unknown:

Height of the cylinder.

Solution:

Curved surface area $2\pi rh = 4.4\ m^2$

$$radius(r) = 0.7m$$

$$height(h) = ?$$

$2\pi rh = 4.4$

$2 \times \frac{22}{7} \times 0.7 \times h = 4.4$

$h = \frac{4.4 \times 7}{2 \times 22 \times 0.7}$

$= 1\ m$

Answer:

The height of the right circular cylinder is $= 1m$.

Q7. The inner diameter of a circular well is 3.5 m. It is 10 m deep. Find

i. Its inner curved surface area

ii. The cost of plastering this curved surface at the rate of 40 per m^2.

Reasoning:

The curved surface area of a right circular cylinder of base radius and height h is $2\pi rh$

Known:

i. The inner diameter and the depth of the well.

ii. Cost of plastering the curved surface area per m^2

i. Its inner curved surface area

Unknown:

The inner curved surface area.

Solution:

Diameter = $2r$ = 3.5 m

$r = \frac{3.5}{2}\ m$

$height = h = 10\ m$

$The\ Curved\ surface\ area\ = 2\pi rh$

$= 2 \times \frac{22}{7} \times \frac{3.5}{2} \times 10$

$= 110\ m^2$

ii. Cost of plastering the curved surface area per m^2

Unknown:

Cost of plastering the curved surface area.

Solution:

Cost of plastering $1m^2 = Rs\ 40$
Cost of plastering $110\ m^2 = 110 \times 40 = Rs\ 4400$

Answer:
Its inner curved surface $= 110\ m^2$
Cost of plastering the curved surface = Rs 4400

Q8. In a hot water heating system, there is a cylindrical pipe of length 28 m and diameter 5 cm. Find the total radiating surface in the system.

Reasoning:

The curved surface area of a right circular cylinder of base radius r and height h is $2\pi rh$.

Known:

The length and diameter of the cylindrical pipe of a water heater.

Unknown:

Radiating surface area in the system.

Solution:

Diameter = 2r = 5cm

$$r = \frac{5}{2}cm = \frac{5}{2 \times 100}\ m$$

$$height = h = 28\ m$$

$$Curved\ \ surface\ area = 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 28 \times \frac{5}{2 \times 100}$$

$$= \frac{22}{5} = 4.4\ m^2$$

Answer:

The total radiating surface $= \frac{22}{5} = 4.4\ m^2$

Q9. Find

i. The lateral or curved surface area of a closed cylindrical petrol storage tank that is 4.2 m in diameter and 4.5 m high.

ii. How much steel was actually used, if 1/12 of the steel actually used was wasted in making the tank?

Reasoning:

The curved surface area of a right circular cylinder of base radius r and height h is $2\pi rh$ and its total surface area is $2\pi r(r+h)$.

Known:

The diameter and height of the storage tank.

i. The lateral or curved surface area

Solution:

$$Diameter \ = \ 2r \ = \ 4.2 \ m$$
$$r = \frac{4.2}{2}$$
$$= 2.1 \ m$$
$$height = h = 4.5 \ m$$
$$Curved \ surface \ area = 2\pi rh$$
$$= 2 \times \frac{22}{7} \times 2.1 \times 4.5$$
$$= 59.4 \ m^2$$

ii. Steel actually used

Solution:

$$Total \ surface \ area \ = 2\pi r(r+h)$$
$$= 2 \times \frac{22}{7} \times 2.1(4.5 + 2.1)$$
$$= 2 \times \frac{22}{7} \times 2.1 \times 6.6$$
$$= 87.12 \ m^2$$

This is the steel used $= 87.12m^2$, $\frac{1}{12}$ of this is wasted.

The area of the steel which has gone into the task $= 1 - \frac{1}{12} = \frac{11}{12}$

$\frac{11}{12}$ of the steel used to make the tank = 87.12

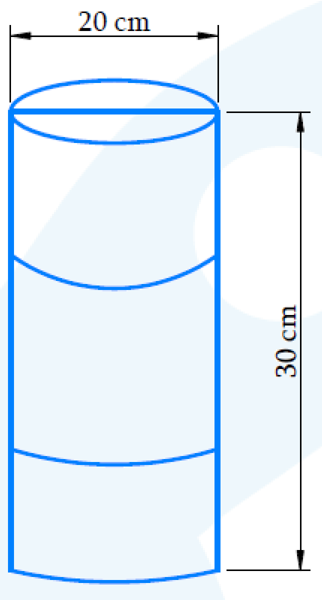
This means actual area of steel used $= \frac{12}{11} \times 87.12 = 95.04\ m^2$

Answer:

Curve surface area $= 59.4 m^2$

Steel actually used $= 95.04 m^2$

Q10. In Fig. 13.12, you see the frame of a lampshade. It is to be covered with a decorative cloth. The frame has a base diameter of 20 cm and height of 30 cm. A margin of 2.5 cm is to be given for folding it over the top and bottom of the frame. Find how much cloth is required for covering the lampshade.



Reasoning:

The curved surface area of a right circular cylinder of base radius r and height h is $2\pi rh$. and its total surface area $2\pi r(r+h)$. The amount of cloth required to cover the cylinder will be equal to surface are of cylinder. (Here height will will equal to the height of cylinder plus 2.5 cm margin on both sides.)

Known:

The height and diameter of the frame of lampshade. And the margin of 2.5m for folding.

Unknown:

Cloth required for covering the lamp shade.

Solution:

Diameter = 2r = 20cm

$\therefore r = 10\ cm$

Height of the lamp shade = 30 cm + 2.5 cm for both side folding
∴ Total height of the cloth = 30 + 2.5 + 2.5 = 35 cm
Cloth required is the curved surface area= $2\pi rh$

$$= 2 \times \frac{22}{7} \times 10 \times 35$$
$$= 2200\ cm^2$$

Answer:
Cloth required $= 2200\ cm^2$

Q11. The students of a Vidyalaya were asked to participate in a competition for making and decorating penholders in the shape of a cylinder with a base, using cardboard. Each penholder was to be of radius 3 cm and height 10.5 cm. The Vidyalaya was to supply the competitors with cardboard. If there were 35 competitors, how much cardboard was required to be bought for the competition?

Reasoning:

The curved surface area of a right circular cylinder of base radius r and height h is $2\pi rh$. And total surface area $= 2\pi r(r+h)$. So, total amount of cardboard required will be the product of surface area of pen holders and total number of participating students.

Known:

Radius and height of the card board.

Unknown:

Card board required to be bought for 35 competitors.

Solution:

$radius(r) = 3\ cm$
$height(h) = 10.5\ cm$
Card board required for 1 competitor = Total surface area of the cylinder
Since the pen holder is open at the top.
One area of πr^2 should be subtracted.
∴ Cardboard required for 1 competitor $= 2\pi rh + \pi r^2$

$$= 2 \times \frac{22}{7} \times 3 \times 10.5 + \frac{22}{7} \times 3 \times 3$$
$$= 198 + \frac{198}{7} = 198(1 + \frac{1}{7})$$
$$= \frac{198 \times 8}{7}\ cm^2$$

Cardboard required for 35 competitions:

$$= \frac{198 \times 8}{7} \times 35 = 7920\ cm^2$$

Answer:

$7920\ cm^2$ of cardboard was required to be bought for the competition.

Exercise 13.3 (Page 221 of Grade 9 NCERT Textbook)

Assume $\pi = \frac{22}{7}$ unless stated otherwise

Q1. Diameter of the base of a cone is 10.5 cm and its slant height is 10 cm. Find its curved surface area.

Reasoning:

Curved surface area of a right circular cone of base radius *r* and slant height *l* is πrl.

Known:

Diameter of the base and slant height.

Unknown:

Curved surface area.

Solution:

Diameter = 2r = 10.5 cm

$\therefore r = \frac{10.5}{2} cm$

Slant height $l = 10\ cm$

Curved surface area $= \pi rl$

$= \frac{22}{7} \times \frac{10.5}{2} \times 10$

$= 165\ cm^2$

Answer:

Curved surface area of the cone $= 165\ cm^2$

Q2. Find the total surface area of a cone, if its slant height is 21 m and diameter of its base is 24 m.

Reasoning:

The total surface area of the cone is the sum of the curved surface area and area of the base which is a circle.

Known:

Diameter of the box and slant height of the cone.

Unknown:

Total surface area of the cone.

Solution:

Total surface area of the cone is $= \pi rl + \pi r^2 = \pi r(l + r)$

Where l is slant height and radius r.

Diameter = 2r = 24 m

r = 12 m

l = 21 m

T S A $= \pi r(l + r)$

$$= \frac{22}{7} \times 12 \times (12 + 21)$$

$$= \frac{22}{7} \times 12 \times 33$$

$$= \frac{8712}{7} = 1244.57\ m^2$$

Answer:

Total surface area of the cone $= 1244.57\,m^2$

Q3. Curved surface area of a cone is $308\ cm^2$ and its slant height is 14 cm. Find

i. Radius of the base

ii. Total surface area of the cone.

Reasoning:

The total surface area of the cone is the sum of the curved surface area and its base area which is a circle.

Known:

Curved surface area of the cone and its slant height.

Unknown:

(i) Radius of the base.

Solution:

Curved surface area= $\pi rl = 308\ cm^2$

Slant height (l) = 14cm

$$\pi rl = 308$$
$$\frac{22}{7} \times r \times 14 = 308$$
$$r = \frac{308}{14} \times \frac{7}{22} = 7\ cm$$

(ii) Total surface area of the cone.

Solution:

Total surface area $= \pi r(l + r)$

Radius = 7 cm

Slant height (l) = 14 cm

$$TSA = \pi r(r + l)$$
$$= \frac{22}{7} \times 7 \times (7 + 14) = 22 \times 21 = 462\ cm^2$$

Answer:

Radius of the cone = 7 cm

Total surface area of the cone $= 462\,cm^2$

Q4. A conical tent is 10 m high and the radius of its base is 24 m. Find

i. Slant height of the tent.

ii. Cost of the canvas required to make the tent, if the cost of 1 m^2 canvas is Rs. 70.

Reasoning:

Curved surface area of the cone of base radius r and slant height l is πrl. Where, $l = \sqrt{r^2 + h^2}$ using the Pythagoras Theorem. And cost of canvas required will be the product of area and cost per meter square of canvas.

Known:

Height of the cone and its base radius.

Unknown:

i. Slant height of the tent.

Solution:

Slant height $l = \sqrt{r^2 + h^2}$
Radius (r) = 24 m
Height (h) = 10 m

$$l = \sqrt{r^2 + h^2}$$
$$l = \sqrt{(24)^2 + (10)^2}$$
$$= \sqrt{576 + 100}$$
$$= \sqrt{676}$$

Slant height of the conical tent $= 26\ m$

ii. Cost of canvas required to make if 1 m^2 canvas is Rs 70.

Canvas required to make the tent is equal to the curved surface area of the cone.
Curved surface area of the cone $= \pi rl$
Radius (r) = 24 m
Slant height $(l) = 26\ m$

$$CSA = \frac{22}{7} \times 24 \times 26\ m^2$$

$\therefore$ Cost of $\frac{22}{7} \times 24 \times 26 m^2$ canvas $= \frac{22}{7} \times 24 \times 26 \times 70 = Rs\ 137280$

Cost of the canvas required to make the tent = Rs. 137280

Answer:

(i) Slant height of the tent is 26 m.
(ii) The cost of the canvas is Rs. 137280.

Q5. What length of tarpaulin 3 m wide will be required to make conical tent of height 8 m and base radius 6 m? Assume that the extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm (Use π = 3.14)

Reasoning:

Curved surface area of a right circular cone of base radius r and slant height is πrl and $l = \sqrt{r^2 + h^2}$

Known:

Height and radius of the cone width of the tarpaulin and its margin.

Unknown:

Length of the tarpaulin.

Solution:

Radius (r) = 6 m
Height (h) = 8 m
Slant height

$$l = \sqrt{r^2 + h^2}$$
$$= \sqrt{6^2 + 8^2}$$
$$= \sqrt{36 + 64}$$
$$= \sqrt{100}$$
$$= 10\ m$$

Curved surface area $= \pi rl$

$$= \pi \times 6 \times 10$$
$$= 3.14 \times 6 \times 10$$
$$= 188.4\ m^2$$

Curved surface area = Area of the tarpaulin

Tarpaulin area $= length \times width$
Width of the tarpaulin = 3 m

$$length \times width = 188.4$$
$$length \times 3 = 188.4$$
$$length = \frac{188.4}{3}$$
$$= 62.8\ m$$

Extra length of the material = 20 cm = 0.2 m
Actual length required = 62.8 +0.2 = 63 m

Answer:
Length of the tarpaulin required = 63 m

Q6. The slant height and base diameter of a conical tomb are 25 m and 14 m respectively. Find the cost of white washing its curved surface at the rate of Rs 210 per 100 m^2.

Reasoning:

Curved surface area of a right circular cone of r base radius and slant height is πrl.

Known:

Slant height and base diameter rate of white washing Rs 210 per 100 m^2.

Unknown:

Cost of white washing its curved surface area.

Solution:

Slant Height (l) = 25 m

$r = \frac{14}{2} = 7m$

Curved surface area $= \pi rl$

$= \frac{22}{7} \times 7 \times 25$

$= 550\ m^2$

Cost of white washing per $100\ m^2 = 210$

For $550\ m^2 = \frac{210 \times 550}{100}$

$=$ Rs 1155

Answer:

Cost of white washing the conical tomb is Rs. 1155.

Q7. A joker's cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 10 such caps.

Reasoning:

Curved surface area of a right circular cone of base radius r and slant height l is πrl and $l = \sqrt{r^2 + h^2}$

Known:

i. Caps box radius and height.
ii. Number of caps.

Unknown:

Area of the sheet required to 10 caps.

Solution:

Base Radius (2r) = 7cm
Height h = 24cm
Slant height $l = \sqrt{r^2 + h^2}$

$= \sqrt{7^2 + 24}$

$= \sqrt{49 + 576}$

$= \sqrt{625}$

$= 25\ cm$

Curved surface area of cone = Surface area of the sheet required for 1cap πrl.

$= \frac{22}{7} \times 7 \times 25$

$= 550\ m^2$

Curved surface area for 10 caps $= 550 \times 10 = 5500\ m^2$

Answer:

The area of the sheet required to make 10 such caps is $5500\ m^2$.

Q8. A bus stop is barricaded from the remaining part of the road, by using 50 hollow cones made of recycled cardboard. Each cone has a base diameter of 40 cm and height 1 m. If the outer side of each of the cones is to be painted and the cost of painting is Rs 12 per m^2. What will be the cost of painting all these cones?
(Use π = 3.14 and take $\sqrt{1.04} = 1.02$)

Reasoning:

Curved surface area of a right circular cone of base radius r and slant height l is πrl and $l = \sqrt{r^2 + h^2}$

Known:

Base diameter and height of the cone number of cones and cost per m^2.

Unknown:

Cost of painting the 50 cones.

Solution:

Base diameter = 40 cm
2r = 40 cm

$r = \frac{40}{2} = 20\ cm$

$= 0.2\ m \quad [\because 100\ cm = 1\ m]$

Height h = 1 m

$$l = \sqrt{r^2 + h^2}$$
$$= \sqrt{(0.2)^2 + (1)^2}$$
$$= \sqrt{1.04}$$
$$= 1.02\,m$$

Curved surface are = πrl

$$= \pi \times 0.2 \times 1.02$$
$$= 3.14 \times 0.2 \times 1.02$$
$$= 0.64056\ m^2$$

Curved surface area of 50 cones

$$= 0.64056 \times 50$$
$$= 32.028\ m^2$$

Cost of painting per m^2 = Rs 12

Cost of painting $= 32.028m^2 = Rs\ 32.028 \times 12$

$$= Rs\ \ 384.34\ m^2$$

Answer:

Cost of painting all the cones =Rs 384.34

Exercise 13.4 (Page 225 of Grade 9 NCERT Textbook)

Assume $\pi = \frac{22}{7}$ unless stated otherwise

Q1. Find the surface area of a sphere of radius:
(i) 10.5 cm (ii) 5.6 cm (iii) 14 cm

Reasoning:

Surface area of sphere of radius r = $4\pi r^2$

Known:

Radius of the sphere.

Unknown:

Surface area of the sphere

Solution:

i. Radius = 10. 5 cm

Surface area of the sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (10.5)^2$$
$$= 1386 \ cm^2$$

ii. Radius = 5.6 cm

Surface area of the sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (5.6)^2$$
$$= 394.24 \ cm^2$$

iii. Radius = 14 cm

Surface area of the sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (14)^2$$
$$= 2464\ cm^2$$

Answer:

Surface area

(i) $1386\ cm^2$

(ii) $394.24\ cm^2$

(iii) $2464\ cm^2$

Q2. Find the surface area of a sphere of diameter:

i.14 cm ii. 21 cm iii. 3.5 m

Difficulty Level: Medium

Reasoning:

Surface area of the sphere of radius r is equal 4 times the area of the circle of radius r. $S = 4\pi r^2$.

Known:

Diameter of the sphere.

Unknown:

Surface area of the sphere.

Solution:

i.

$$Diameter\ (2r) = 14\ cm$$
$$Radius\,(r) = 7\ cm$$
$$Surface\ area = 4\pi r^2 = 4 \times \frac{22}{7} \times 7 \times 7$$
$$= 616\ cm^2$$

ii.

$$Diameter\ (2r) = 21\,cm$$

$$Radius(r) = \frac{21}{2}\,cm$$

$$\text{Surface area} = 4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$$
$$= 1386\ cm^2$$

iii.

$$Diameter\ (2r) = 3.5\ m$$

$$Radius\ (r) = \frac{3.5}{2}\ m$$

$$\text{Surface area} = 4 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}$$
$$= 38.5\ m^2$$

Answer: Surface areas are-

(i) 616 cm^2

(ii) 1386 cm^2

(iii) 38.5 m^2

Q3. Find the total surface area of a hemisphere of radius 10 cm. (Use π = 3.14)

Reasoning:

A hemisphere is half of a sphere having one circular surface at the top.
∴ Total surface area of a hemisphere is the half area of sphere and the top circular area $= 3\pi r^2$

Known:

Radius of the hemisphere.

Unknown:

Total surface area of the hemisphere.

Solution:

Radius of the hemisphere (r) = 10 cm

Total Surface area $= 3\pi r^2$
$= 3 \times 3.14 \times 10 \times 10$
$= 942\ cm^2$

Answer:
Total surface area of the hemisphere $= 942\ cm^2$

Q4. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.

Reasoning:

The surface area of the sphere is $= 4\pi r^2$

Known:

The radius of balloon.

Unknown:

Ratio of the surface area.

Solution:

$$\frac{CSA_1}{CSA_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = (\frac{7}{14})^2 = (\frac{1}{2})^2 = \frac{1}{4}$$

Answer:
The ratio of the surface areas of the balloon $= 1:4$

Q5. A hemispherical bowl made of brass has inner diameter 10.5 cm. Find the cost of Tin-plating it on the inside at the rate of 16 per 100 m^2 .

Reasoning:

Hemisphere is half of a sphere so surface area is $\frac{4\pi r^2}{2} = 2\pi r^2$.

Known:

Ratio of tin plating per $100\ m^2$ and inner diameter.

Unknown:

Cost of tin planting

Solution:
Inner Diameter (2r) = 10.5 cm

Inner Radius $r = \frac{10.5}{2}$

Surface area = $2\pi r^2$

$$= 2 \times \frac{22}{7} \times \frac{10.5}{2} \times \frac{10.5}{2}$$
$$= 173.25\ cm^2$$

Cost of plating is 16 per $100\ cm^2$

$\therefore$ Cost of plating for $173.25\ cm^2$

$$= \frac{173.25 \times 16}{100}$$
$$= Rs\ \mathbf{27.72}$$

Answer:

Cost of tin plating = Rs 27.72

Q6. Find the radius of a sphere whose surface area is $154\, m^2$.

Reasoning:

The surface area of the sphere is equal to 4 times the surface area of the circle if r is the sphere and circle. $S = 4\pi r^2$

Known:

Surface area of the sphere.

Unknown:

Radius of the sphere.

Solution:

Surface area of the sphere $= 4\pi r^2$

$$154 = 4 \times \frac{22}{7} \times r^2$$
$$r^2 = \frac{154}{4} \times \frac{7}{22}$$
$$r^2 = 7 \times 7 \times \frac{1}{2} \times \frac{1}{2}$$
$$r = 7 \times \frac{1}{2} = 3.5\ cm$$

Answer:

Radius of the sphere = 3.5 cm

Q7. The diameter of the moon is approximately one fourth of the diameter of the earth. Find the ratio of their surface areas.

Reasoning:

Surface of the sphere having radius r is equal to the 4 times are of the circle having radius r. $S = 4\pi r^2$

Known:

Ratio between the diameter of the moon and earth.

Unknown:

Ratio of the surface areas.

Solution:

Let the diameter of the earth be 2r.

Radius of the earth in = r

Radius of the moon is $= \frac{r}{4}$

Surface area of the earth = $4\pi r^2$

Surface area of the moon $= 4\pi(\frac{r}{4})^2$

$\therefore$ Ratio of their surface area $= \frac{\text{Surface area of the moon}}{\text{Surface area of the earth}}$

$$= \frac{4 \times \pi \times r^2}{4 \times \pi \times 4 \times 4 \times r^2} = \frac{1}{16}$$

Answer:
Ratio of their surface area $= 1:16$

Q8. A hemispherical bowl is made of steel, 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl.

Reasoning:

The surface area of the hemisphere is half of the surface area of the sphere.

Known:

Thickness of steel.

Unknown:

Outer curved surface area.

Solution:

Inner radius bowl = 5 cm
Thickness of steel = 0.25 cm
∴ Outer radius of the bowl = 5+0.25
=5.25 cm
Curve Surface area of the hemisphere = $2\pi r^2$

$$= 2 \times \frac{22}{7} \times (5.25)^2$$

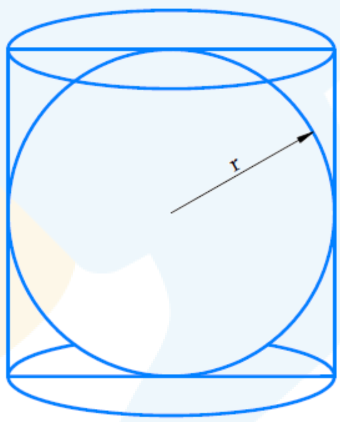
$$= 173.25\ cm^2$$

Answer:

The outer curved surface area of the hemisphere $= 173.25\ cm^2$

Q9. A right circular cylinder just encloses a sphere of radius r (see Fig. 13.22). Find

i. Surface area of the sphere
ii. Curved surface area of the cylinder
iii. Ratio of the areas obtained in (i) and (ii).



Reasoning:

The Curved surface area of the cylinder is given by $2\pi rh$,and Surface area of the sphere is given by $4\pi r^2$

Known:

Radius of the sphere which touches the cylinder.

i. Surface area of the sphere

Solution:

Radius of the sphere in = r
Surface area $=4\pi r^2$

i. Curved surface area of the cylinder.

Solution:

For cylinder: radius $= r$
Height =2r
Curved surface area of the cylinder $= 2\pi rh$

$$= 2\times\pi\times r\times 2r$$
$$= 4\pi r^2$$

iii. Ratio of the areas obtained in (i) and (ii).

Ratio of the area = $\dfrac{\text{Surface area of sphere}}{\text{Curved surface area of cylinder}}$

$$= \frac{4\pi r^2}{4\pi r^2} = \frac{1}{1} = 1:1$$

Answer:

i. Surface area of the sphere $= 4\pi r^2$
ii. Curved surface area of cylinder $= 4\pi r^2$
iii. Ratio between (i) and (ii) $= 1:1$

Exercise 13.5 (Page 228 of Grade 9 NCERT Textbook)

Q1. A matchbox measures 4 cm × 2.5 cm × 1.5 cm. What will be the volume of a packet containing 12 such boxes?

Reasoning:

Volume of cuboid is $base\ area \times height$

$\therefore volume = length \times breadth \times height$

Volume is the space occupied by the solid.

Known:

The measurements l, b, h of a cuboid.

Unknown:

Volume of the container containing 12 such boxes.

Solution:

$length(l) = 4\ cm$

$breadth(b) = 2.5\ cm$

$height(h) = 1.5\ cm$

Volume of a cuboid $= l \times b \times h$

Volume of 1 matchbox $= 4 \times 2.5 \times 1.5$

Volume of 12 such boxes $= 4 \times 2.5 \times 1.5 \times 12 \quad = 180\ cm^3$

Answer:

Volume of a packet containing 12 such boxes $= 180\ cm^3$

Q2. A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many liters of water can it hold? ($1\ m^3 = 1000$ l)

Reasoning:

Capacity of the tank is nothing but the volume of the cuboid.

Known:

The length, breadth and height the tank.

Unknown:

Capacity of the tank in litres.

Solution:

Capacity of the tank = Volume of the cuboidal tank.
Length $(l) = 6\ m$
Breadth $(b) = 5\ m$
Height $(h) = 4.5\ m$
Volume of a cuboid $= 6 \times 5 \times 4.5 = 135\ m^3$

$$1\ m^3 = 1000\ l$$
$$\therefore 135\ m^3 = 135 \times 1000$$
$$= 135000\ l$$

Answer:

Litres of the water the cuboidal water tank can hold $= 135000\ l$.

Q3. A cuboidal vessel is 10 m long and 8 m wide. How high must it be made to hold 380 cubic metres of a liquid?

Reasoning:

Capacity of the cuboid vessel is the cuboid.
Volume of the cuboid is product of length, breadth and height.

Known:

Length and breadth of the cuboidal vessel, and the capacity of the vessel.

Unknown:

The height of the vessel having $380 m^3$ capacity.

Solution:

Capacity of the cuboidal vessel = volume of the cuboid.
Volume of the cuboid $= l \times b \times h$
$Length(l) = 10\ m$
$Breadth(b) = 8\ m$
$Volume(V) = 380\ cm^3$
$Height(h) = ?$

$$380 = 10 \times 8 \times h$$

$$h = \frac{380}{10 \times 8}$$
$$= \frac{19}{4}$$
=4.75 m

Answer:
The cuboidal vessel must be made 4.75 m height.

Q4. Find the cost of digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of Rs 30 per m^3.

Reasoning:

Volume of the pit will be equal to volume of a cuboid and the cost to dug the pit will be equal to product of the volume of the pit and cost to dig the pit per meter cube.

Known:

The length, breadth and height the cuboid. Cost of digging Rs 30 per m^3 .

Unknown:

Cost of digging the cuboidal pit.

Solution:

$length(l) = 8\ m$
$breadth(b) = 6\ m$
$height(h) = 3\ m$
Volume of the cuboid $= l \times b \times h \ = 8 \times 6 \times 3\ m^3$
Cost of the digging the pit is Rs 30 per m^3
∴ Cost of digging for $8 \times 6 \times 3\ m^3$
$= 30 \times 8 \times 6 \times 3$
$= Rs\ 4320$

Answer:
Cost of digging the cuboidal pit is = Rs 4320.

Q5. The capacity of a cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its length and depth are respectively 2.5 m and 10 m.

Reasoning:

Capacity of the cuboidal tank = Volume of the cuboidal tank.
Volume of the cuboid $= l \times b \times h$

Known:

Capacity of the tank length and height of the tank.

Unknown:

Breadth of the tank.

Solution:

Capacity of the tank = 50000 litres.

$$1000\ l = 1\ m^3$$
$$50000 l = \frac{50000}{1000} = 50\ m^3$$

Volume has to be changed in m^3 because all the measurements are in meter.
Volume of the cuboid $= l \times b \times h$

$$length\ (l) = 2.5\ cm$$
$$breadth\ (b) = ?$$
$$height (h) = 10\ cm$$

$$2.5 \times 10 \times b = 50$$
$$b = \frac{50}{2.5 \times 10}$$
$$= 2\ m$$

Answer:
The breadth of the cuboidal tank is 2 m.

Q6. A village, having a population of 4000, requires 150 litres of water per head per day. It has a tank measuring 20 m × 15 m × 6 m. For how many days will the water of this tank last?

Reasoning:

Capacity of the tank is the volume of cuboid.
Volume of the $= length \times breadth \times height$

Known:

Requirement of water per head per day measurements of the tank.

Unknown:

Number of days the water of the tank will last.

Solution:

Requirement of water per head per day = 150 litres.
Requirement of water for 4000 population $= 150 \times 4000 = 600000 l$

$$1000 l = 1\ m^3$$
$$\therefore 600000 l = \frac{600000}{1000} = 600\ m^3$$

$$length\ of\ the\ tank (l) = 20\ m$$
$$breadth\ of\ the\ tank (b) = 15\ m$$
$$height\ of\ the\ tank (h) = 6\ m$$

Capacity of the tank = Volume of the cuboid

$$= l \times b \times h$$
$$= 20 \times 15 \times 6$$
$$= 1800\ m^3$$

Number of days for which the water of the tank will last

$$= \frac{\text{Capacity of the tank}}{\text{Requirement of water for the total population}}$$
$$= \frac{1800}{600} = 3\ days$$

Answer:
The water of tin tank will last for 3 days.

Q7. A godown measures 40 m × 25 m × 15 m. Find the maximum number of wooden crates each measuring 1.5 m × 1.25 m × 0.5 m that can be stored in the godown.

Reasoning:

The maximum number of wooden crates that can be stored in the godown will be the ratio of volume of the godown to the volume of wooden crate.

Known:

Measurements of the godown measurements of the wooden crates.

Unknown:

Number of crates that can be stored in the godown.

Solution:

Volume of the godown = Capacity of godown

$length(l) = 40\ m$
$breadth(b) = 25\ m$
$height(h) = 15\ m$
Capacity of the godown $= 40 \times 25 \times 15 = 1500\ m^3$

Volume of the wooden crate:

$length(l) = 1.5\ m$
$breadth(b) = 1.25\ m$
$height(h) = 0.5\ m$
Volume $= 1.5 \times 1.25 \times 0.5 \quad = 0.9375\ m^3$

Number of crates = $\dfrac{\text{Capacity of the godown}}{\text{Volume of the crate}} = \dfrac{1500}{0.9375} = 1600$

Answer:
The maximum number of wooden crates that can be stored in the godown is 1600.

Q8. A solid cube of side 12 cm is cut into eight cubes of equal volume. What will be the side of the new cube? Also, find the ratio between their surface areas.

Difficulty Level: Medium

Reasoning:

Volume of the cube is a^3 when the side measurement is a. Surface area is nothing but the sum of the area of the 6 faces.

Known:

Side length of the cube. Number of cubes cut from the bigger cube.

Unknown:

Volume of the small cube and ratio between the surface areas of the bigger and the smaller cube.

Solution:

Side of the solid cube = 12 cm

Volume of the solid cube = $a^3 = (12)^3 = 1728\ m^3$

It is cut in to 8 equal cubes of same volume.

$\therefore$ Volume of one small cube $= \dfrac{1728}{8} \quad = 216 m^3$

Let x be side of the small cube.

$\therefore$ Volume of one small cube $x^3 cm^3$

$$x^3 = 216$$

$$x = \sqrt[3]{216}$$

$$= (216)^{\frac{1}{3}}$$

$$= (6 \times 6 \times 6)^{\frac{1}{3}}$$

$$= 6\ cm$$

For the ratio of this surface area we have to find the surface area of the two cubes having side 12cm and 6 cm respectively.

Surface area of the cube $= 6 \times a^2$

Surface area of the cube having 12 cm side

$= 6 \times 12^2$

Surface area of the cube having 6cm side

$= 6 \times 6^2$

$\therefore ratio = \dfrac{6 \times 12 \times 12}{6 \times 6 \times 6}$

$= \dfrac{4}{1} = 4:1$

Answer:

The side of the new cube is 6 m.

Ratio between the surface area = 4:1.

Q9. A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea in a minute?

Reasoning:

Water that falls in to the sea in the volume of the cuboid.

Known:

Depth, wide and rate of water following per hour.

Unknown:

Amount of water will fall in to the sea in a minute.

Solution:

$$length(l) = 2\ km = 2 \times 1000 = 2000\ m$$
$$breadth(b) = 40\ m$$
$$height(h) = 3\ m$$

Water fall in the sea = Volume of cuboid= lxbxh

$= 2000 \times 40 \times 3$

This is for 1 hour

Therefore, water fell in to the sea in a minute.

$$= \frac{2000 \times 40 \times 3}{60}$$
$$= 4000\ m^3$$

Answer:

$4000\ m^3$ of water will fall into the sea in a minute.

Exercise 13.6 (Page 230 of Grade 9 NCERT Textbook)

Assume $\pi = \frac{22}{7}$ unless stated otherwise

Q1. The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water can it hold? $(1000\ cm^3 = 1\ l)$

Reasoning:

Volume of a cylinder of base radius r and height is V $= \pi r^2 h$.

What is the known/given?

Circumference of the base and the height.

What is the unknown?

Litres of water the cylindrical vessel can hold.

Solution:

Since the base of a cylindrical vessel is a circle, the circumference is $2\pi r = 132\ cm$ (given)

$$2 \times \frac{22}{7} \times r = 132$$
$$r = \frac{132 \times 7}{2 \times 22}$$
$$= 21\ cm$$

Radius $(\mathrm{r}) = 21\mathrm{cm}$

Height $(\mathrm{h}) = 25cm$

Capacity of the cylindrical vessel = volume of the cylindrical vessel $= \pi r^2 h$

$$= \frac{22}{7} \times 21 \times 21 \times 25$$
$$= 34650\ cm^3$$

Answer should be in litres

$$1000\ cm^3 = 1\ l$$

$$34650\ cm^3 = x\ l$$

$$x = \frac{34650}{1000}$$

$$= 34.65\ l$$

Answer:

Capacity of the cylindrical vessel $= 34.65\ l$

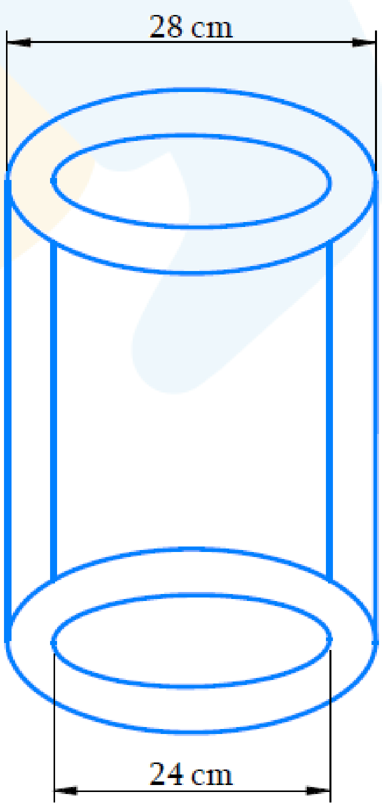
Q2. The inner diameter of a cylindrical wooden pipe is 24 cm and its outer diameter is 28 cm. The length of the pipe is 35 cm. Find the mass of the pipe, if $1\ cm^3$ of wood has a mass of 0.6 g.

Difficulty Level: Medium

Reasoning:

Since the cylindrical wooden pipe is made up of two concentric circles at the top and bottom, we have to find the volume of both the cylinders.

Diagram



What is the known/given?

Diameter of the inner and outer cylinder. Length of the cylinder.

What is the unknown?

Mass of the pipe if 1 cm^3 of pipe has mass 6 g.

Solution:

Volume of the outer cylinder $V_1 = \pi R^2 h$
Outer diameter *(2R) = 28 cm*
Outer radius $(R) = \frac{28}{2} = 14\ cm$
Height (h) = 35 cm
Volume of outer cylinder $V_1 = \frac{22}{7} \times 14 \times 14 \times 35 = 21560\ cm^3$
Volume of inner cylinder $V_2 = \pi r^2 h$
Inner diameter $(2r) = 24\ cm.$
Inner radius $(r) = \frac{24}{2} = 12\ cm$
Height h = 35 cm
Volume of outer cylinder $V_2 = \frac{22}{7} \times 12 \times 12 \times 35 = 15840\ cm^3$
Volume of the wood used = Outer volume – inner volume

$= 21560 - 15840$
$= 5720\ cm^3$

Mass of 1 cm^3 pipe = 0.6 g

$\therefore$ Mass of $5720\ cm^3$ pipe $= 0.6 \times 5720$
$= 3432\ g$
or
$= 3.432\ kg$

Answer: Mass of the pipe = 3.432 kg

Q3. A soft drink is available in two packs – (i) a tin can with a rectangular base of length 5 cm and width 4 cm, having a height of 15 cm and (ii) a plastic cylinder with circular base of diameter 7 cm and height 10 cm. Which container has greater capacity and by how much?

Difficulty Level: Medium

Reasoning:

Volume of the cuboid is lbh and volume of cylinder in $\pi r^2 h$

What is the known/given?

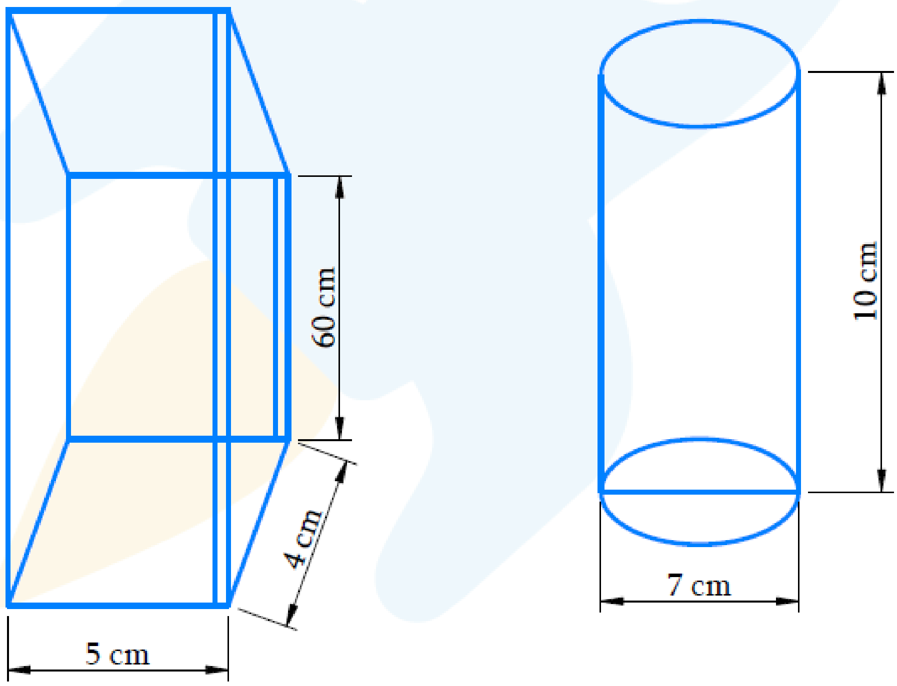
Measurements of cuboid can. Measurements of cylindrical can.

What is the unknown?

Which can is big and how much?

Solution:

Diagram



Tin Can -

Length (l) = 5 cm
Breadth (b) = 4 cm
Height (h) = 15 cm

Capacity = Volume $= lbh$

$= 5 \times 4 \times 15$

$= 300\ cm^3$

Plastic Cylinder –

Diameter (2r) =7 cm

Radius $(r) = \frac{7}{2}$ cm

Height (h) =10 cm

Capacity = Volume $= \pi r^2 h$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 10$

$= 385\ \ cm^3$

Clearly, the plastic cylinder has greater capacity than the tin container.

Difference $= 385 - 300 = 85 cm^3$

Answer:

The plastic cylindrical can have more capacity than the Tin Can by $= 85\ cm^3$.

Q4. If the lateral surface of a cylinder is 92.4 cm^2 and its height is 5 cm, then find

(i) radius of its base

(ii) its volume. (Use π = 3.14)

Difficulty Level: Medium

Reasoning:

Lateral surface area of a cylinder is $2\pi rh$.

Volume of cylinder is $\pi r^2 h$.

What is the known/given?

Lateral surface area of the cylinder and its height.

What is the unknown?

Base radius.

Solution:

Lateral surface area of a cylinder having r as radius and height h is $2\pi rh$

Lateral surface area $= 94.2 cm^2$

Height h = 5 cm

$$2\pi rh = 94.2$$

$$2 \times 3.14 \times r \times 5 = 94.2$$

$$r = \frac{94.2}{5 \times 3.14 \times 2} = 3\ cm$$

What is the unknown?

Volume of the cylinder.

Solution:

Radius r = 3 cm
Height h = 5 cm

Volume of cylinder $= \pi r^2 h = 3.14 \times 3 \times 3 \times 5 = 141.3 cm^3$

Answer:

Radius of its base = 3cm
Its volume $= 141.3\ cm^3$

Q5. It costs Rs 2200 to paint the inner curved surface of a cylindrical vessel 10 m deep. If the cost of painting is at the rate of Rs 20 per m^2, find.

(i) inner curved surface area of the vessel,
(ii) radius of the base,
(iii) capacity of the vessel.

Reasoning:

The ratio between the total cost and the rate per m^2 will give the inner curve surface area. Curve surface area of a cylinder is $2\pi rh$ if the height of the cylinder is h and base radius is r.

What is the known/given?

Total cost to paint and cost per m^2. And height of the cylindrical vessel.

What is the unknown?

Inner curved surface area.

Solution:

Total cost to paint =2200

Cost of paint per m^2 =20

$$\therefore \text{ Surface area } = \frac{2200}{200} = 110\ cm^2$$

What is the unknown?

Radius of the base.

Solution:

Inner curved surface $= 110\ cm^2$

Height (h) = 10m

$$2\pi rh = 110$$
$$2 \times \frac{22}{7} \times r \times 10 = 110$$
$$r = \frac{110 \times 7}{22 \times 2 \times 10}$$
$$= \frac{7}{4}$$
$$= 1.75\ cm$$

What is the unknown?

Capacity of the vessel.

Solution:

Capacity of the vessel = Volume of the vessel = $\pi r^2 h$

Radius (r) $= 1.75\ cm$

Height (h) $= 10\ cm$

$$\text{Capacity} = 2 \times \frac{22}{7} \times 1.75 \times 1.75 \times 10$$
$$= 96.25\ cm^3$$

The capacity of the vessel is $= 96.25\ cm^3$

Answer:

Inner curved surface area $110\ cm^2$
Radius of the box =1.75 m
Capacity of the vessel $= 96.25\ cm^3$

Q6. The capacity of a closed cylindrical vessel of height 1 m is 15.4 litres. How many square metres of metal sheet would be needed to make it?

Reasoning:

Surface area total of a closed cylinder is $2\pi rh + 2\pi r^2$.
$2\pi rh$ corresponds to the Curved Surface Area of the cylinder and $2\pi r^2$ corresponds to the area of the circular lids of the cylindrical vessel in order to make it a closed container.
Volume of cylinder $\pi r^2 h$

What is the known/given?

Capacity and height of the cylinder.

What is the unknown?

Metal sheet needed to make it.

Solution:

Capacity = 15.4 liters

$$V = \frac{15.4}{1000} = .0154\ m^3$$

$$\begin{aligned} \text{Volume } &= \pi r^2 h \\ 0.0154 &= \frac{22}{7} \times r^2 \times 1 \\ r^2 &= \frac{.0154 \times 7}{22} \\ r^2 &= .0049 \\ r &= \sqrt{.0049} \\ &= 0.07\ m \end{aligned}$$

Curved surface area $= 2\pi rh + 2\pi r^2 = 2\pi r(r+h)$

$$= 2 \times \frac{22}{7} \times 0.07 \times (0.07 + 1) = 0.4708\ m^2$$

Answer:

$0.47\, m^2$ of metal sheet would be needed.

Q7. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled in the interior. The diameter of the pencil is 7 mm and the diameter of the graphite is 1 mm. If the length of the pencil is 14 cm, find the volume of the wood and that of the graphite.

Difficulty Level: Medium

Reasoning:

Volume of cylinder is given by $\pi r^2 h$.

What is the known/given?

Diameter of the pencil, diameter of graphite and length of the pencil.

What is the unknown?

Volume of the wood and that if graphite.

Solution:

For cylinder graphite.

Diameter (2r) = 1 mm

Radius $(r) = \frac{1}{2}\ mm$

Length of the pencil (h) = 14 cm = 140 mm

Capacity = Volume = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times 140$$

$$= 110\ mm^3$$

In $cm^3 = \frac{110}{10 \times 10 \times 10} = 0.11\ cm^3$

For cylinder of wood:

To find the volume of the wood:

Total volume of the pencil - Volume of graphite

$$\pi R^2 h - \pi r^2 h$$
$$\pi h(R^2 - r^2)$$

Diameter of pencil (2R) = 7 mm

Radius $(r) = \frac{7}{2} mm$

Length of the pencil (h) = 14 cm = 140 mm

Volume of wood = $\pi h(R^2 - r^2)$

$$= \frac{22}{7} \times 140 \times [(\frac{7}{2})^2 - (\frac{1}{2})^2]$$
$$= \frac{22}{7} \times 140 \times [\frac{49}{4} - \frac{1}{2}]$$
$$= 5280\ mm^3$$
$$= \frac{5280}{10 \times 10 \times 10} cm^3$$
$$= 5.28\ cm^3$$

Answer:

Volume of the wood $= 5.28\ cm^3$

Volume of graphite $= 0.11\ cm^3$

Q8. A patient in a hospital is given soup daily in a cylindrical bowl of diameter 7 cm. If the bowl is filled with soup to a height of 4 cm, how much soup the hospital has to prepare daily to serve 250 patients?

Reasoning:

Volume of cylinder $\pi r^2 h$. So, the amount of soup to be prepared will be the product of volume of the bowl (with h = 4cm) and total number of patients.

What is the known/given?

Diameter of the bowl and length of the soup, number of patients**.**

What is the unknown?

Total soup to be prepared.

Solution:

Volume of soup bowl $= \pi r^2 h$

Diameter (2R) = 7 cm

Radius $(r) = \frac{7}{2}\ cm$

Height (h) = 4 cm

$\therefore$ Volume of soup in one bowl $= \pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 4$$

$$= 154\ cm^3$$

Volume of soup for 1 patient $= 154\ cm^3$

Volume of soup for 250 patients

$$= 154 \times 250$$

$$= 38500\ cm^3$$

Answer:

Volume of soup to be prepared for 250 patients is $= 38500\ cm^3$ or 38.5 litres [as 1000 cm^3 = 1 litre]

Exercise 13.7 (Page 233 of Grade 9 NCERT Textbook)

Assume $\pi = \dfrac{22}{7}$ unless stated otherwise

Q1. Find the volume of the right circular cone with

(i) radius 6 cm, height 7 cm (ii) radius 3.5 cm, height 12 cm

Reasoning:

Volume of the cone $= \dfrac{1}{3}\pi r^2 h$

Where r is base radius and h is height.

What is the known/given?

Radius and height of two cones.

What is the unknown?

Volume of the cone.

Solution:

Radius (r) = 6 cm
Height (h) = 7 cm

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 7$$
$$= 264\ cm^3$$

Radius (r) = 3.5 cm
Height (h) = 12 cm

$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 12$$
$$= 154\ cm^3$$

Answer:

Volume of the cone $= 264\ cm^3$

Volume of the cone $= 154\ cm^3$

Q2. Find the capacity in litres of a conical vessel with

(i) radius 7 cm, slant height 25 cm (ii) height 12 cm, slant height 13 cm

Reasoning:

Capacity of a conical vessel is nothing but the Volume of the cone $=\frac{1}{3}\pi r^2 h$

What is the known/given?

Radius (r) = 7 cm
Slant height (1) = 25 cm

What is the unknown?

Capacity of the vessel in litres.

Solution:

Capacity of the conical vessel= volume of cone $=\frac{1}{3}\pi r^2 h$

Where $h=\sqrt{l^2-r^2}$
Radius $(r)=7cm$
Slant height $(l)=25cm$

$$\therefore h=\sqrt{(25)^2-7^2}$$
$$=\sqrt{(25+7)(25-7)} \;[\,(a^2-b^2)=(a+b)(a-b)]$$
$$=\sqrt{32\times 18}$$
$$=\sqrt{2\times 2\times 2\times 2\times 2\times 3\times 3\times 2}$$
$$=\sqrt{576}$$
$$=24\ cm$$

Volume $=\frac{1}{3}\times\frac{22}{7}\times 7\times 7\times 24=1232\ cm^3\ or\ 1.232l$

$[\because 1000\ cm^3=1l]$

i.

What is the known/given?

Radius (r) = 12 cm
Slant height $(l)=13\ cm$

What is the unknown?

Capacity of the vessel in litres.

Solution:

Capacity of the conical vessel= volume of cone $= \frac{1}{3}\pi r^2 h$

Height (h) = 12 cm
Slant height $(l) = 13\ cm$

$$r^2 + h^2 = l^2$$
$$r^2 = l^2 - h^2$$
$$r = \sqrt{l^2 - h^2}$$
$$= \sqrt{(13)^2 - (12)^2}$$
$$= \sqrt{(13+12)(13-12)}$$
$$= \sqrt{25}$$
$$= 5\ cm$$

Capacity $= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 12 = \frac{2200}{7} = \frac{2200}{7000} = \frac{11}{35} l$

Answer:

Capacity =1.232 L

Capacity $= \frac{11}{35}$ L

Q3. The height of a cone is 15 cm. If its volume is $1570\ cm^3$, find the radius of the base. (Use π = 3.14).

Reasoning:

Volume of the cone is $\frac{1}{3}$ times of the volume of a cylinder having radius r and height h i.e. $\frac{1}{3}\pi r^2 h$.

What is the known/given?

Volume of the cone and the height of the cone.

What is the unknown?

Radius of the base.

Solution:

Height (h) = 15 cm
Radius (r) = ?

Volume of cone = $\frac{1}{3}\pi r^2 h$

$$1570 = \frac{1}{3}\pi r^2 h$$

$$1570 = \frac{1}{3} \times 3.14 \times r^2 \times 15$$

$$r^2 = \frac{1570 \times 3}{15 \times 3.14} = \frac{4710}{47.1}$$

$$r^2 = 100$$

$$r = \sqrt{100} = 10\ cm$$

Answer:

Radius of the base = 10 cm

Q4. If the volume of a right circular cone of height 9 cm is $48\pi\ cm^3$, find the diameter of its base.

Reasoning:

Volume of the cone is $\frac{1}{3}$ time of the volume of a cylinder having radius r and height h $\frac{1}{3}\pi r^2 h$.

What is the known/given?

Volume and height of the cone.

What is the unknown?

Base diameter.

Solution:

Height (h) = 9 cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$48\pi = \frac{1}{3}\pi r^2 h$$

$$48\pi = \frac{1}{3}\pi \times r^2 \times 9$$

$$r^2 = \frac{48\times 3}{3\times 3} = \frac{48\times 3}{3\times 3} = 16$$

$$r = \sqrt{16} = 4\ cm$$

Base diameter $= 2r = 2\times 4 = 8\ cm$

Answer:

The diameter of the box of the night circular cone is 8 cm.

Q5. A conical pit of top diameter 3.5 m is 12 m deep. What is its capacity in kiloliters?

Reasoning:

Volume of the cone is $\frac{1}{3}$ times of the volume of a cylinder having radius r and height h $= \frac{1}{3}\pi r^2 h$.

What is the known/given?

On a meter and the depth of the cone.

What is the unknown?

Volume in kilometer.

Solution:

Volume of cone $= \frac{1}{3}\pi r^2 h$

Height of the cone (h) = depth of the pit.
Height (h) = 12 cm
Diameter = 2r = 3.5 m

$$\therefore r = \frac{3.5}{2}$$

Volume $= \frac{1}{3}\times\frac{22}{7}\times(1.75)^2\times 12 = 38.5\ cm^3$

$1\ cm^3 = 1000\ l = 1\ k\ l$

$\therefore 38.5\ cm^3 = 38500\ l = 38.5\ kl$

Answer:

Capacity in kilo-litre $= 38.5\ kl$.

Q6. The volume of a right circular cone is $9856\,cm^3$. If the diameter of the base is 28 cm, find

(i) height of the cone (ii) slant height of the cone

(iii) curved surface area of the cone

Reasoning:

Volume of the cone is $\frac{1}{3}$ times of the volume of a cylinder having radius r and height h $= \frac{1}{3}\pi r^2 h$. Curved surface area of the cone is πrl

What is the known/given?

Volume of the right circular cone and diameter of the base.

i.

What is the unknown?

Height of the cone.

Solution:

Radius (r) = 28/2 cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$9856 = \frac{1}{3} \times \frac{22}{7} \times \frac{28}{2} \times \frac{28}{2} \times h$$

$$h = \frac{9856 \times 3 \times 7 \times 2 \times 2}{22 \times 28 \times 28}$$

$$= 48\ cm$$

ii.

What is the unknown?

Slant height of the cone.

Solution:

Radius (r) = 14 cm
Height (h) = 48 cm

$$l = \sqrt{r^2 + h^2}$$
$$= \sqrt{14^2 + 48^2}$$
$$= \sqrt{196 + 2304}$$
$$= \sqrt{2500}$$
$$= 50 \ cm$$

iii.

What is the unknown?

Curved surface area of the cone.

Solution:

Radius (r) = 14 cm
Slant height (l) = 50 cm
Curved surface area $= \pi rl$

$$= \frac{22}{7} \times 14 \times 50$$
$$= 2200 \ cm^2$$

Answer:

Height of the cone = 48cm
Slant height of the cone = 50cm
Surface area $= 2200 \ cm^2$

Q7. A right triangle ABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Reasoning:

Volume of the cone is $\frac{1}{3}$ times of the volume of a cylinder $= \frac{1}{3}\pi r^2 h$.

What is the known/given?

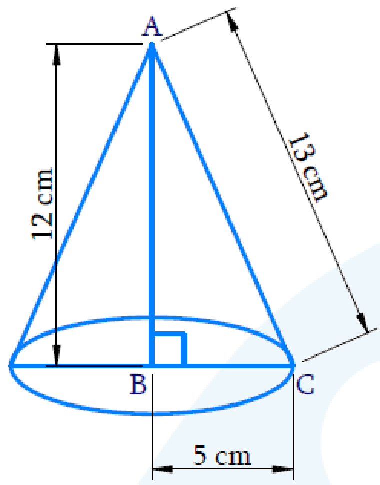
Sides of the right triangle.

What is the unknown?

Volume of the solid obtained.

Solution:

Diagram



Radius (r) = 5 cm
height (h) = 12 cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 12$$

$$= \frac{2200}{7} \text{ or } 100\pi$$

Answer:

Volume of the cone $100\pi\ cm^3$.

Q8. If the triangle ABC in the Question 7 above is revolved about the side 5 cm, then find the volume of the solid so obtained. Find also the ratio of the volumes of the two solids obtained in Questions 7 and 8.

Reasoning:

Volume of the right circular cone is $\frac{1}{3}$ time of the volume of a cylinder $= \frac{1}{3}\pi r^2 h$.

What is the known/given?

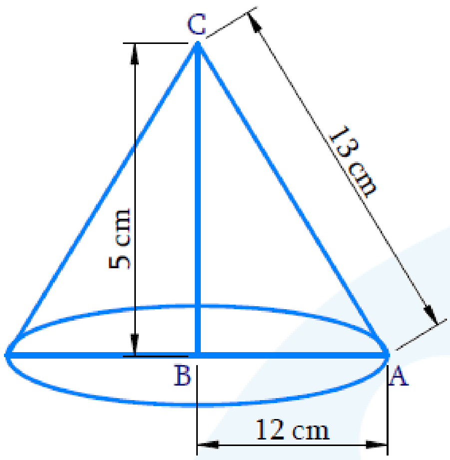
Sides of the right triangle.

What is the unknown?

Volume of the cone and ratio between the volumes.

Solution:

Diagram



Radius of the cone = 12 cm
Height of the cone = 5 cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times (12)^2 \times 5$

$= 240\pi \ cm^3$

Volume in question $7 = 100\pi \ cm^3$

Ratio $100:240 = 5:12$

Answer:

Volume of the cone is $= 240\pi \ cm^3$

Ratio between the volume= $5:12$

Q9. A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume. The heap is to be covered by canvas to protect it from rain. Find the area of the canvas required.

Reasoning:

Volume of the right circular cone is $\frac{1}{3}$ times of the volume of a cylinder. Curved surface area of the cone $= \pi r l$

What is the known/given?

Diameter and height of the cone.

What is the unknown?

Volume of the cone and area of the canvas to cover the heap.

Solution:

Diameter = 2r = 10.5 m

$$r = \frac{10.5}{2} = 5.25m$$

Height (h) = 3 m

$$\text{Volume of cone } = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times (\frac{10.5}{2})^2 \times 3$$
$$= 86.625 \ m^3$$

$$\text{Slant height } l = \sqrt{r^2 + h^2}$$
$$= \sqrt{(5.25)^2 + (3)^2}$$
$$= \sqrt{27.5625 + 9}$$
$$= \sqrt{36.5625}$$
$$= 6.05 \ m$$

The area of the canvas to cover the heap of wheat = surface area of the cone = πrl

$$= \frac{22}{7} \times \frac{10.5}{2} \times 6.05$$
$$= 99.825 \ m^2$$

Answer:

Volume of the cone $= 86.625 \ m^3$

Area of the canvas required $= 99.825 \ m^2$

Exercise 13.8 (Page 236 of Grade 9 NCERT Textbook)

Q1. Find the volume of a sphere whose radius is

(i) 7 cm (ii) 0.63 m

Reasoning:

Volume of the sphere $= \frac{4}{3}\pi r^3$

What is the known/given?

Radius of the spheres.

What is the unknown?

Volume of the sphere.

Solution:

i. Radius = 7 cm

$$\text{Volume of the sphere} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times 7^3$$
$$= 1437.33\ cm^3$$

ii. Radius = 0.63 m

$$\text{Volume of the sphere} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times (0.63)^3$$
$$= 1.05\ m^3$$

Answer:

Volume (i) $= 1437.33\ cm^3$

Volume (ii) $= 1.05\ m^3$

Q2. Find the amount of water displaced by a solid spherical ball of diameter.

(i) 28 cm (ii) 0.21 m

Reasoning:

The amount of water displaced by a solid spherical ball is nothing but its volume.

Volume of the sphere $= \frac{4}{3}\pi r^3$

What is the known/given?

Diameter of the sphere.

What is the unknown?

Amount of water displaced by the solid.

Solution:

i. Diameter $= 2r = 28\ cm$

Radius $r = \frac{28}{2} = 14\ cm$

Amount of water displaced by the solid sphere = Volume of the sphere.

$$= \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times (14)^3$$
$$= 11498.66\ cm^3$$

ii. Diameter $= 2r = 0.21\ m$

Radius $r = \frac{0.21}{2} = 0.105\ m$

Amount of water displaced by the solid sphere = Volume of the sphere.

$$= \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times (0.105)^3$$
$$= .004851\ m^3$$

Answer:

i. Amount of water displaced $= 11498.66\ cm^3$

ii. Amount of water displaced $= .004851\ m^3$

Q3. The diameter of a metallic ball is 4.2 cm. What is the mass of the ball, if the density of the metal is 8.9 g per cm^3?

Reasoning:

Volume of the sphere $= \frac{4}{3}\pi r^3$ and mass is the product of density times volume.

What is the known/given?

Diameter of the sphere and density of the metal per m^3 .

What is the unknown?

Mass of the ball.

Solution:

Diameter $= 2r = 4.2\ cm$

Radius $r = \frac{4.2}{2} = 2.1\ cm$

Volume of the sphere

$$= \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times (2.1)^3$$
$$= 38.808\ cm^3$$

Density = 8.9 g per cm^3

Mass $= volume \times density$
$$= 38.808 \times 8.9$$
$$= 345.39\, g$$

Answer:

Mass of the ball $= 345.39\, g$

Q4. The diameter of the moon is approximately one-fourth of the diameter of the earth. What fraction of the volume of the earth is the volume of the moon?

Reasoning:

Volume of the sphere $= \frac{4}{3}\pi r^3$. So, the fraction of the volume of the earth is the volume of the moon is the ratio of the volume of the earth to the volume of the moon

What is the known/given?

Ratio between the diameters of moon and earth.

What is the unknown?

Fraction of the volume of the earth is the volume of the moon

Solution:

Diameter of the Earth = 2r

Radius of the Earth $= \frac{1}{2}(2r) = r$

Diameter of moon $= \frac{1}{4}(2r) = \frac{1}{2}r$

Radius of the moon $= \frac{1}{2}(\frac{r}{2}) = \frac{r}{4}$

Volume of the earth $(v_1) = \frac{4}{3}\pi r^3$

Volume of the moon $(v_2) = \frac{4}{3}\pi(\frac{r}{4})^3$

$$= \frac{1}{64}(\tfrac{4}{3}\pi r^3)$$

$$= \frac{1}{64} \text{ Volume of earth}$$

Answer:

Hence the volume of the moon is $\frac{1}{64}$ th fraction of the volume of the earth.

Q5. How many liters of milk can a hemispherical bowl of diameter 10.5 cm hold?

Reasoning:

Volume of the hemisphere $= \frac{2}{3}\pi r^3$

What is the known/given?

Diameter of the bowl.

What is the unknown?

Quantity of milk which the hemispherical bowl can hold.

Solution:

Diameter $= 2r = 10.5\ cm$

$$r = \frac{10.5}{2} = 5.25$$

Amount of water displaced

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times (5.25)^3$$

$$= 303.19\ cm^3$$

$$= 0.303 l \qquad [\because 1000\ cm^3 = 1\ l]$$

Answer: 0.303 litres of milk can be held in the bowl.

Q6. A hemispherical tank is made up of an iron sheet 1 cm thick. If the inner radius is 1m, then find the volume of the iron used to make the tank.

Reasoning:

The volume of the hemisphere is given by $\frac{2}{3}\pi r^3$ where r will be equal to the summation of thickness of sheet and inner radius of tank.

What is the known/given?

Inner radius and thickness of the iron sheet**.**

What is the unknown?

Volume of the iron used.

Solution:

Inner radius $(r) = 1\ m$

Thickness of the sheet $= 1\ cm = .01\ m$

Outer radius $(R) = inner\ radius + thickness$

$$= 1\ m + 0.01\ m = 1.01\ m$$

Volume of the iron used to make the tank

$$= \frac{2}{3}\pi(R^3 - r^3)$$

$$= \frac{2}{3} \times \frac{22}{7} \times [1.01^3 - 1^3]$$
$$= .06348\ m^3$$

Answer:

Volume of the iron used $= .06348\ m^3$.

Q7. Find the volume of a sphere whose surface area is $154\ cm^2$.

Reasoning:

Surface area of the sphere = $4\pi r^2$

Volume of the sphere $= \frac{4}{3}\pi r^3$

What is the known/given?

Surface area of the sphere.

What is the unknown?

Volume of the sphere.

Solution:

Let the radius of the sphere be r.

Surface area of the sphere $= 4\pi r^2$

Surface area $= 154\ cm^2$

$$4\pi r^2 = 154$$
$$r^2 = \frac{154}{4} \times \frac{7}{22}$$
$$r^2 = \frac{49}{4}$$
$$r = \frac{7}{2}\ cm$$

Volume of the sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (\frac{7}{2})^3$$
$$= \frac{539}{3}\ cm^3$$

$$= 179\frac{2}{3}\ cm^3$$

Answer:

Volume of the sphere $= 179\frac{2}{3}\ cm^3$

Q8. A dome of a building is in the form of a hemisphere. From inside, it was white washed at the cost of Rs 4989.60. If the cost of white washing is Rs2 per square meter, find the

(i) inside surface area of the dome,
(ii) Volume of the air inside the dome.

Reasoning:

Surface area of the sphere= $4\pi r^2$

Volume of the sphere $= \frac{4}{3}\pi r^3$

What is the known/given?

Total cost and per square meter.

What is the unknown?

Inside surface area of the done.
Volume of the width the doom.

Solution:

Total cost for white wash = Rs 4989.6

Cost per $m^2 = 2.00$

$\therefore$ Inside surface area $= \frac{4989.6}{2} = 2494.8\ m^2$

Let the radius of the hemisphere be r.
Inside surface area $= 2494.8\ m^2$

$$2\pi r^2 = 2494.8$$

$$2\times\frac{22}{7}\times r^2 = 2494.8$$
$$r^2 = \frac{2494.8\times 7}{2\times 22}$$
$$r^2 = 396.9$$
$$r = \sqrt{396.9}$$
$$r = 19.9\ m$$

Volume of the air inside the doom = Volume of the hemisphere $=\frac{2}{3}\pi r^3$

$$=\frac{2}{3}\times\frac{22}{7}\times(19.9)^3$$
$$=16511.73\ m^3$$

Answer:

Inner surface area of the doom =2494.8 m^2

Volume of the air inside the doom =16511.73 m^3

Q9. Twenty-seven solid iron spheres, each of radius r and surface area S are melted to form a sphere with surface area S′. Find the

(i) Radius r′ of the new sphere,
(ii) Ratio of S and S′.

Reasoning:

Volume of the sphere $=\frac{4}{3}\pi r^3$

What is the known/given?

Number of spheres melted.

What is the unknown?

Radius.

Ratio of S and S'.

Solution:

Volume of a solid iron sphere $= \frac{4}{3}\pi r^3$

Volume of 27 solid spheres $= 27(\frac{4}{3}\pi r^3) = 36\pi r^3$

Volume of new sphere $= 36\pi r^3$

Let the radius of the new sphere = r'

Volume of the new sphere $= \frac{4}{3}\pi r'^3$

According to the question,

$$\frac{4}{3}\pi r'^3 = 36\pi r^3$$

$$r'^3 = \frac{(36r^3)(3)}{4}$$

$$= 27r^3$$

$$r' = \sqrt[3]{(27r^3)}$$

$$r' = 3r$$

Radius of the new sphere r' $= 3r$

$$S = 4\pi r^2$$

$$s^1 = 4\pi(3r)^2$$

$$\frac{s}{s^1} = \frac{4\pi r^2}{4\pi(3r)^2}$$

$$\frac{s}{s^1} = \frac{1}{3^2} = \frac{1}{9}$$

So ratio of S:S' is 1:9

Answer:

Radius r of new sphere = 3r.

Ratio of S and S'= 1:9

Q10. A capsule of medicine is in the shape of a sphere of diameter 3.5 mm. How much Medicine (in mm^3) is needed to fill this capsule?

Reasoning:

Volume of the sphere $= \frac{4}{3}\pi r^3$

What is the known/given?

Diameter of the capsule.

What is the unknown?

Medicine needed to fill the capsule in mm^3 .

Solution:

Volume of the sphere $= \frac{4}{3}\pi r^3$

Diameter = 2r = 3.5 mm

Radius $= r = \frac{3.5}{2} = 1.75 mm$

Capacity of capsule

$$= \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times (1.75)^3$$

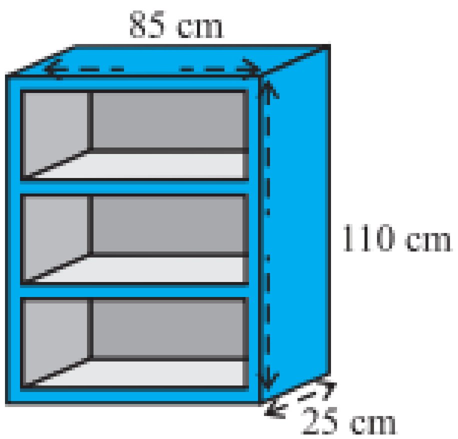
$$= 22.46\ mm^3$$

Answer:

22.46 mm^3 of medicine is needed to fill this capsule.

Exercise 13.9(Page 236 of Grade 9 NCERT Textbook)

Q1. A wooden bookshelf has external dimensions as follows: Height = 110 cm, Depth = 25 cm, Breadth = 85 cm (see Fig. 13.31). The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2. Find the total expenses required for polishing and painting the surface of the bookshelf.



Difficulty Level: Hard

Reasoning:

Dimensions of the cupboard and thickness wood.

What is the known/given?

Rate for polishing and painting.

What is the unknown?

Total expenses for painting and polishing

Solution:

$$length(l) = 25\ cm$$

$$breadth(b) = 85\ cm$$

$$height(h) = 110\ cm$$

Surface area to be polished $= (h \times b) + 2(h \times l) + 2(b \times l) + 2(h \times 5) + 4(75 \times 5)$
$= (110 \times 85) + 2(110 \times 25) + 2(85 \times 25) + 2(110 \times 25) + 4(75 \times 5)$
$= (9350 + 5500 + 4250 + 1100 + 1500)cm^2$
$= 21700cm^2$

Expense required for polishing at the rate of 20 paise per cm^2

$\therefore$ Total expense $= \dfrac{21700 \times 20}{100}$
$= \text{Rs } 4340$

Surface area to be painted
$= [2(20 \times 90) + 6(75 \times 20) + (75 \times 90)]$
$= (3600 + 9000 + 6750)cm^2$
$= 19350cm^2$

At the rate of 10 paise per sq. cm = 1935

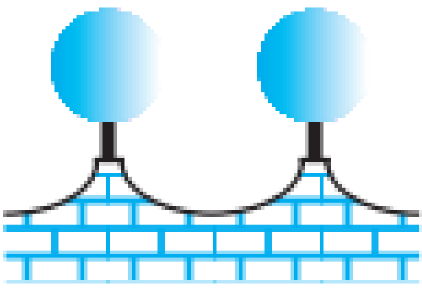
Expense required for polishing and painting the surface of the bookshelf.

$= 4340 + 1935 = 6275$

Answer:

Total expense required for polishing and painting the surface of the bookshelf = 6275.

Q2. The front compound wall of a house is decorated by wooden spheres of diameter 21cm, placed on small supports as shown in Fig 13.32. Eight such spheres are used for this purpose and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2.



Difficulty level: Hard

Reasoning:

Surface area of sphere $= 4\pi r^2$
Surface area of the cylinder $= 2\pi rh$

What is the known/given?

Diameter of the sphere.
Measurement of the cylinder.
Cost per cm^2 for silver and black paint.

What is the unknown?

Cost of the paint.

Solution:

For wooden sphere:

Diameter = 2r = 21 mm

$$= r = \frac{21}{2}\ cm$$

Surface area for wooden sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times \left(\frac{21}{2}\right)^2$$

$$= 1386\ cm^2$$

Since the support is in cylinder of radius 1.5cm.

So the area of wooden sphere to be painted $= 1386 - \frac{22}{7}(1.5)^2$ [Area of the sphere is πr^2]

$$= 1378.93\ m^2$$

Total number of sphere are 8.

Total volume of the sphere to be Silver painted $= 8 \times 1378.93$

$$= 11031.44\ cm^2$$

Cost of painting at the rate of 25 paise per m^2.

$$= \frac{11031.44 \times 25}{100}$$

$$= 2757.86$$

For a cylindrical support.

Radius (r) =1.5 m

Height $(h) = 7\ m$

Surface of the cylindrical support= $2\pi rh$

$$= \frac{2}{3} \times \frac{22}{7} \times 1.5 \times 7 = 66\ m^2$$

Surface area of 8 cylindrical support

$$= 8 \times 66 = 528\ cm^2$$

Cost of black painting per $cm^2 = 5$paise

$$\text{Total cost of black painting} = \frac{528 \times 5}{150}$$

$$= 26.40$$

Cost of paint $= 2757.856 + 26.40$

$$= 2784.26$$

Answer:

Cost of paint = 2784.26

Q3. The diameter of a sphere is decreased by 25%. By what percent does its curved surface area decrease?

Difficulty Level: Hard

Reasoning:

Curved surface area of a sphere is $4\pi r^2$. So, if diameter is decreased by 25%, then radius also decreases by 25% so, the percentage change in curved surface area will be the ratio of difference between old area and new area by old area multiplied by 100.

What is the known/given?

Decreased percentage of diameter.

What is the unknown?

Decreased percentage of curved surface area.

Solution:

Let the radius of the sphere be $\frac{r}{2}$ cm.

Then it's diameter $= 2(\frac{r}{2})$ = r cm

Curved surface area of the original sphere:

$$= 4\pi\left(\frac{r}{2}\right)^2 = \pi r^2 cm^2$$

New diameter of the sphere:

$$r = r - r \times \frac{25}{100} \text{ [}diameter\ of\ \ a\ sphere\ is\ decreased\ by\ 25\%\text{]}$$

$$= \frac{3r}{4} cm$$

Radius of the new sphere:

$$= \frac{1}{2}(\frac{3r}{4}) = \frac{3}{8r} cm$$

New curved surface area of the sphere.

$$= 4\pi(\frac{3r}{8})^2 = \frac{9\pi r^2}{16} cm^2$$

Decrease in the original curved Surface area $= \pi r^2 - \frac{9\pi r^2}{16}$

$$= \frac{16\pi r^2 - 9\pi r^2}{16}$$

$$= \frac{7\pi r^2}{16}$$

Percentage of decrease in the original curved surface area

$$= \frac{7\pi r^2}{16} \times 100$$

$$= 43.75\ \%$$

Answer:

Hence the original curved surface area decrease by $= 43.75\%$