Similar Triangles

Review Exercise Answers

Level-1

S1. (D). using the BPT, we have:

$$\frac{AD}{DB}=\frac{AE}{EC}$$
$$\Rightarrow \frac{2.5cm}{3\,cm}=\frac{3.75\,cm}{EC}$$
$$\Rightarrow EC = 4.5cm$$

Thus,

$$AC = AE + EC = 8.25$$

S2. (C). using the BPT in ΔBEA , we have:

$$\frac{BF}{FE}=\frac{BD}{DA} \quad \text{...(i)}$$

Using the BPT in ΔBCA we have

$$\frac{BD}{DA}=\frac{BF}{EC} \quad \text{...(ii)}$$

From (i) and (ii),

$$\frac{BF}{FE}=\frac{BE}{EC}$$

S3. Using the BPT in ΔCDB gives

$$\frac{CF}{FD}=\frac{CE}{EB}$$

Using the BPT in ΔCAB gives

$$\frac{CD}{DA}=\frac{CE}{EB}$$

Thus,

$$\frac{CF}{FD}=\frac{CD}{DA}$$
$$\Rightarrow \frac{CF}{CF+FD}=\frac{CD}{CD+DA} \quad \text{(how?)}$$
$$\Rightarrow \frac{CF}{CD}=\frac{CD}{AC}$$
$$\Rightarrow CD^2 = CF \times AC$$

S4. (A). using the angle bisector theorem in ΔADB, we have

$$\frac{AD}{DB}=\frac{AE}{EB} \quad ...(i)$$

Similarly, in ΔADC, we have

$$\frac{AD}{DC}=\frac{AF}{FC} \quad ...(ii)$$

Using (i) and (ii) and the fact that DB=DC, we have:

$$\frac{AE}{EB}=\frac{AF}{FC}$$

Thus $EF \parallel \mathrm{BC}$ and this is true regardless of what kind of triangle ΔABC is.

S5. (B) since $\angle APB=\angle DPC$ (vertically opposite angles), and $\angle A=\angle D=90^\circ$, we conclude that $\angle APB \sim \angle DPC$ by the AA criterion. Thus,

$$\frac{AP}{DP}=\frac{BP}{CP}$$

$$\Rightarrow AP\times PC=BP\times PD$$

S6. We note that $\Delta ABX \sim \Delta ACY$ (both are equilateral), and thus,

$$\frac{area(\Delta ABX)}{area(\Delta ACY)}=\frac{AB^2}{AC^2}=\frac{AB^2}{\left(\sqrt{2}AB\right)^2}=\frac{1}{2}$$

S7. (D). we note that $\Delta BDE \sim \Delta BAC$, and thus,

$$\frac{area(\Delta BDE)}{area(\Delta BAC)}=\frac{BD^2}{BA^2}=\frac{BD^2}{(BD+DA)^2}$$

$$=\frac{1}{\left(1+\frac{DA}{BD}\right)^2}=\frac{1}{\left(1+\frac{3}{2}\right)^2}=\frac{4}{25}$$

Thus,

$$\frac{area(trap\ ADEC)}{area(\Delta ABC)}=1-\frac{area(\Delta BDE)}{area(\Delta BAC)}$$

$$=1-\frac{4}{25}=\frac{21}{25}$$

S8. (C). comparing ΔABD and ΔABC we note that $\angle A=\angle A$, and $\angle ADB=\angle ABC$, and thus by the AA criterion

$$\Delta ADB \sim \Delta ABC$$

$$\frac{area(\Delta ADB)}{area(\Delta ABC)}=\frac{AB^2}{AC^2}$$

$$=\frac{AC^2-BC^2}{AC^2}=\frac{144}{169}$$

S9. (B) we have (using the Pythagoras theorem):

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= AB^2 + 4BD^2 \\ &= AB^2 + 4\left(AD^2 - AB^2\right) \\ &= AB^2 + 4AD^2 - 4AB^2 \\ &= 4AD^2 - 3\,AB^2 \end{aligned}$$

Multiple options may be correct

S10. All the four options are correct. In ΔABO, we use the BPT to obtain:

$$\frac{AD}{DB} = \frac{AE}{EO}$$

Similarly, in ΔAOC, we have $\Delta AEF \sim \Delta AOC$ and thus,

$$\frac{AF}{AC} = \frac{EF}{OC}$$

Now, since

$$\frac{AD}{DB} = \frac{AE}{EO} \quad and \quad \frac{AF}{FC} = \frac{AE}{EO}$$

We have

$$\frac{AD}{DB} = \frac{AF}{AC}$$

Thus, $DF \parallel \text{BC}$ and hence $\Delta ADF \sim \Delta ABC$, which also means that

$$\frac{AD}{DB} = \frac{AF}{AC}$$

S11. Again, all four options are correct. Self-exercise.

S12. (B), (C) and (D). Using the BPT, we have:

$$\frac{AF}{FB} = \frac{AE}{EC} \qquad \frac{BG}{GA} = \frac{BD}{DC} \qquad \ldots\text{(i)}$$

We also note that

$$\begin{aligned} AF = BG &\Rightarrow FB = GA \\ &\Rightarrow \frac{AF}{FB} = \frac{BG}{GA} \end{aligned}$$

From (i) and (ii), we have

$$\frac{AE}{EC} = \frac{BD}{DC} \Rightarrow ED \parallel AB$$

Only option (A) is incorrect

$$\left(\frac{AE}{EC} \text{ equals } \frac{AF}{FB}, \text{ not } \frac{AF}{AB}\right).$$

S13. (B) and (D). Using the angle bisector theorem in ΔOAB, ΔOBC, ΔOAC, we have

$$\frac{OA}{OB}=\frac{AF}{FB}\Rightarrow OA.FB=OB.AF$$
$$\frac{OB}{OC}=\frac{BD}{DC}\Rightarrow OB.CD=OC.BD$$
$$\frac{OC}{OA}=\frac{CE}{EA}\Rightarrow OC.AE=OA.CE$$

Also,

$$\frac{OA}{OB}\times\frac{OB}{OC}\times\frac{OC}{OA}=\frac{AF}{FB}\times\frac{BD}{DC}\times\frac{CF}{EA}$$
$$\Rightarrow 1=\frac{AF.BD.CE}{BF.CD.AE}$$
$$\Rightarrow AF.BD.CE=BF.CD.AE$$

S14. (C) and (D). Comparing ΔBAC *and* ΔADC we have:

$$\angle BAC=\angle ADC,\ \angle BCA=\angle ACD$$

Thus, $\Delta BAC\sim\Delta ADC$ by the AA criterion (option (B) is incorrect because the order of the vertices in the similarity relation is incorrect). Now, we have:

$$\frac{AB}{AD}=\frac{BC}{AC}=\frac{AC}{CD}$$
$$\Rightarrow\frac{AB}{BC}=\frac{AD}{AC},\ AC^2=BC\times CD$$

We see that (C) and (D) are correct.

S15. (A), (B) and (C). We have:

$$(6x)^2+(8x)^2=100x^2=(10x)^2$$
$$(8(x-1))^2+(15(x-1))^2$$
$$=(64+225)(x-1)^2$$
$$=289(x-1)^2$$
$$=(17(x-1))^2$$
$$(2x-1)^2+\left(2\sqrt{2x}\right)^2$$
$$=4x^2-4x+1+8x$$
$$=4x^2+4x+1$$
$$=(2x+1)^2$$

On the other hand,

$$x^2+(x+2)^2$$
$$=x^2+x^2+4x+4$$
$$=2x^2+4x+4$$

is not equal to $(x+5)^2$ in general.

S16. (B), (C) and (E). we note that $\Delta ABD \sim \Delta BCD \sim \Delta ACB$, and so:

$$\frac{b}{c}=\frac{AD}{b}=\frac{p}{a}, \qquad ...(i)$$

$$\frac{b}{a}=\frac{p}{CD}=\frac{AD}{p} \qquad ...(ii)$$

Thus, from (i), we have:

$$cp = ab$$
$$\Rightarrow c^2p^2 = a^2b^2$$
$$\Rightarrow (a^2+b^2)p^2 = a^2b^2$$
$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}$$

Also, from (ii),

$$p^2 = AD \times CD$$

Integers answer

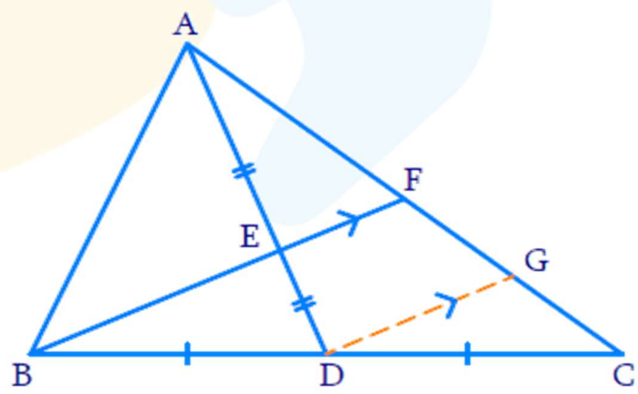
S17. We have:

$$\frac{AD}{DB}=\frac{AE}{EC}=\frac{3}{5}$$
$$\Rightarrow \frac{AE}{AC}=\frac{3}{3+5}=\frac{3}{8}$$
$$\Rightarrow AE=\frac{3}{8}\times AC = 1.8cm$$

Thus,

$5AE = 5\times 1.8cm = 9cm$

S18. Through D, draw $DG \parallel BF$, as shown below:



In ΔCFB, we apply the converse of the mid-point theorem to conclude that CG=GF.
Similarly, in ΔADG, we can conclude that GF=FA.

Thus, AF=FG=GC

This means that.

$$AF = \frac{1}{3}AC \Rightarrow \frac{AC}{AF} = 3$$

S19. Let BD = x cm. Then, CD will be (12 - x) cm. Using the angle bisector theorem, we have:

$$\frac{AB}{AC} = \frac{BD}{CD} \Rightarrow \frac{10}{6} = \frac{x}{12-x}$$

$$\Rightarrow 120 - 10x = 6x$$

$$\Rightarrow x = \frac{120}{16} = 7.5$$

Thus, $$2BD = 2 \times 7.5 = 15cm.$$

S20. We note that $\Delta ABC \sim \Delta YZX$ by the AA criterion. Let us now prove that the ratios of sides of two Similar triangle are the same as the ratio of their perimeters. Let a, b, c be the side length, in one triangle, and A, B, C be the side lengths in the second (similar) triangle. We have:

$$\frac{a}{A} = \frac{b}{B} = \frac{c}{C} = \frac{a+b+c}{A+B+C}$$

$$= \frac{p}{p}$$

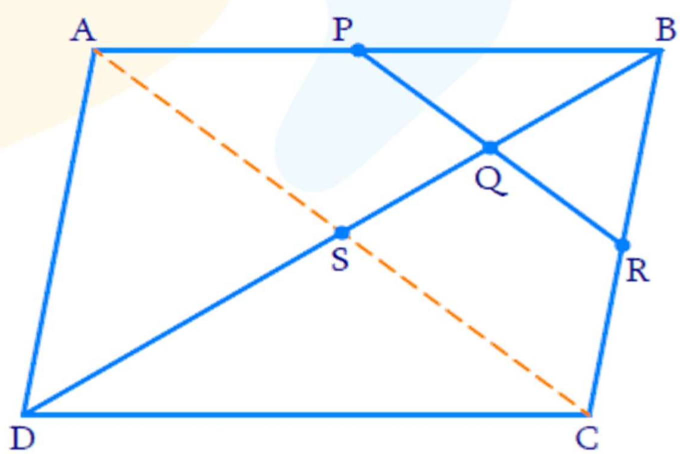
Where p and P are the two perimeter values.
Thus, in our present scenario,

$$\frac{AC}{XY} = \frac{perimeter(\Delta ABC)}{perimeter(\Delta XYZ)}$$

$$= \frac{20cm}{15cm} = \frac{4}{3}$$

$$\Rightarrow XY = \frac{3}{4}AC = 6cm$$

S21. Consider the following figure:



Note that

$$BQ = \frac{1}{4}BD = \frac{1}{2}BS$$

Thus, Q is the mid-point of BS which means that $PQ \parallel AS$ by the mid-point theorem.

Now, in ΔBAC, P is the mid-point of BA and $PR \parallel AC$. Thus, R is the mid-point of BC, or $\frac{BR}{RC} = 1.$

S22. Since $DE \parallel BC$, $\Delta ADE \sim \Delta ABC$, and so,

$$\frac{DE}{BC} = \frac{AD}{AB} = \frac{AD}{AD + DB} = \frac{4}{10}$$

$$\Rightarrow BC = \frac{10}{4} \times DE = \frac{10}{4} \times 6$$

$$= 15\,cm$$

S23. Using similarity between ΔBCD *and* ΔABD, we can prove that

$$BD^2 = AD \times CD$$

Thus,

$$AD = \frac{BD^2}{CD} = 9\,cm$$

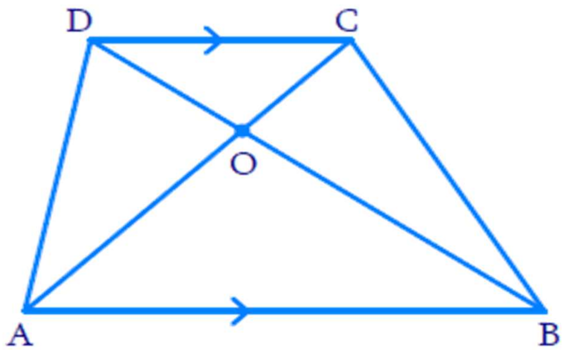
S24. We know that ratio of area of two similar triangle is the square of the ratio of the sides of the triangles. Thus, in the case, the ratio of the sides will be

$$\sqrt{\frac{100cm^2}{64cm^2}} = \frac{5}{4}$$

This will also be the ratio of the altitudes. Thus, the required altitude length l is given by:

$$\frac{10\,cm}{l} = \frac{5}{4} \Rightarrow l = 8\,cm$$

S25. Consider the following figure:



Clearly, $\Delta AOB \sim \Delta COD$ and thus,

$$\frac{area(\Delta AOB)}{area(\Delta COD)} = \left(\frac{AB}{CD}\right)^2 = 4$$

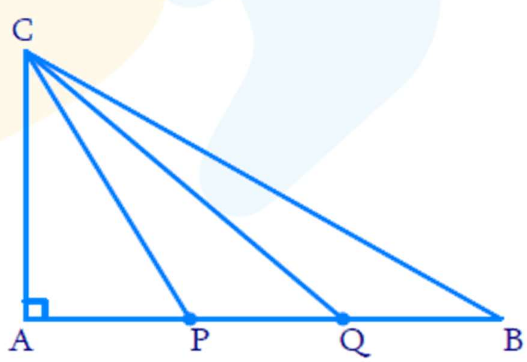
$$\Rightarrow area(\Delta COD) = \frac{1}{4} area(\Delta AOB)$$

$$= 25\,cm^2$$

S26. It is easy to prove that $\Delta ABC \sim \Delta DEF$, and so:

$$\frac{area(\Delta ABC)}{area(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = 4$$

S27. Consider the following figure:



We have:

$$CB^2 = CA^2 + BA^2$$
$$= CA^2 + (3AP)^2$$
$$= CA^2 + 9AP^2$$
$$CP^2 = CA^2 + AP^2$$

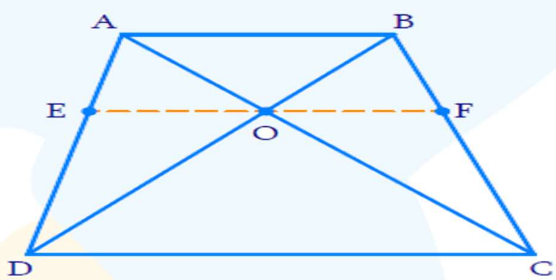
Thus,

$$3CB^2 + 5CP^2 = 3(CA^2 + 9AP^2) + 5(CA^2 + AP^2)$$
$$= 8CA^2 + 32AP^2$$
$$= 8(CA^2 + 4AP^2)$$
$$= 8(CA^2 + (2AP)^2)$$
$$= 8(CA^2 + AQ^2)$$
$$= 8CQ^2$$

This means that $\lambda = 8$.

Miscellaneous

S28. (a) Consider the following figure:



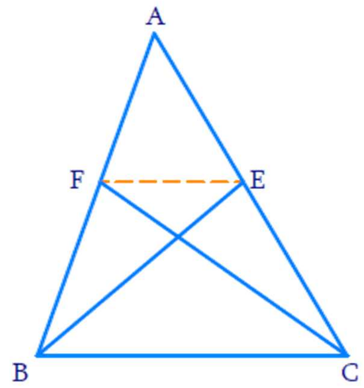
Through O, we have drawn EF || AB || DC. We make the following observations, using the basic proportionality theorem in each case:

- In $\Delta ABC, \dfrac{AO}{OC} = \dfrac{BF}{FC}$
- $\Delta BDC, \dfrac{BO}{OD} = \dfrac{BF}{FC}$

Comparing these two relations, we conclude that AO : OC = BO : OD.

(b) This part is left to you as an exercise.

S29. Consider the following figure, and compare ΔFBC and ΔEBC:

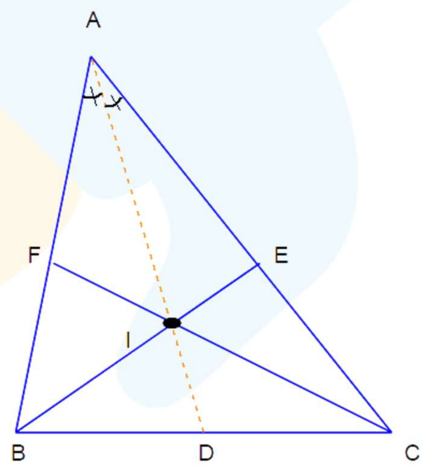


We have:

- $BC = BC$
- $\angle B = \angle C$
- $\angle FCB = \angle EBC\left(=\frac{1}{2}\angle B \text{ or } \frac{1}{2}\angle C\right)$

By the ASA criterion, $\Delta FBC \cong \Delta EBC$, so that FB = FC. Also, because AB = AC, this means that AF = AE. We thus conclude that AF: FB = AE: EC. The BPT now implies that FE || BC.

S30. Consider the following figure, where we have also drawn the angle bisector AD of $\angle A$, which will pass through I:

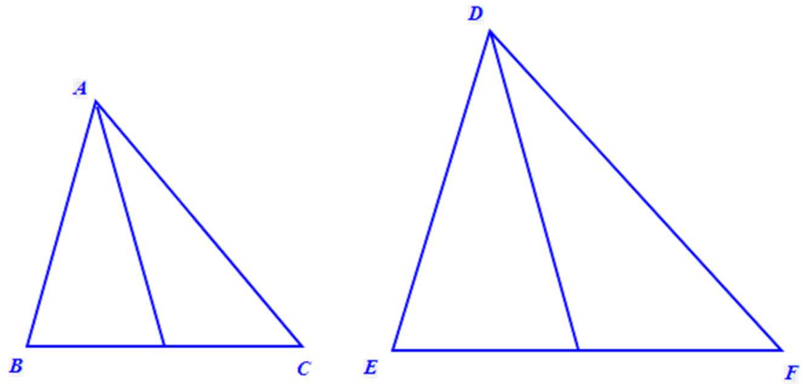


Applying the angle bisector theorem to $\angle AFC$, we have:

$$\frac{AF}{AC}=\frac{FI}{CI} \Rightarrow \frac{AF}{FI}=\frac{AC}{CI}$$

This completes our proof.

S31. Consider the following figure:



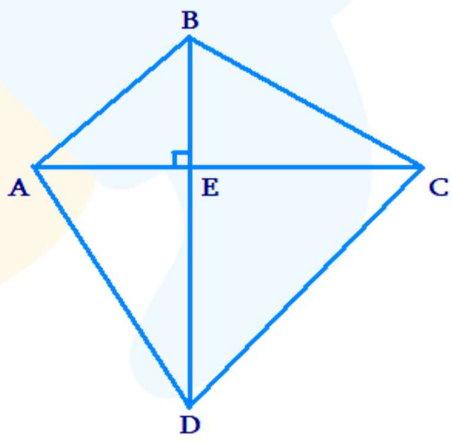
We observe that since $\Delta ABC \sim \Delta DEF,$ we have

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{BC/2}{EF/2} = \frac{BX}{EY} \qquad \ldots(1)$$

By the SAS criterion for similarity, $\Delta ABX \sim \Delta DEY,$ so that $\frac{AB}{DE} = \frac{AX}{DY}$. Using this and (1), we conclude that $\frac{AX}{DY} = \frac{BC}{EF}$.

This completes our proof.

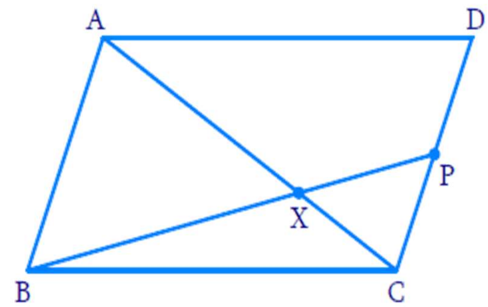
S32. Consider the following figure, which shows AC and BD intersecting at E at right angles:



Using the Pythagoras Theorem, we have:

$$AB^2 = AE^2 + BE^2$$
$$CD^2 = CE^2 + DE^2$$
$$\Rightarrow AB^2 + CD^2 = AE^2 + BE^2 + CE^2 + DE^2$$
$$= \left(BE^2 + CE^2\right) + \left(AE^2 + DE^2\right)$$
$$= BC^2 + AD^2$$

S33. Consider the following figure:



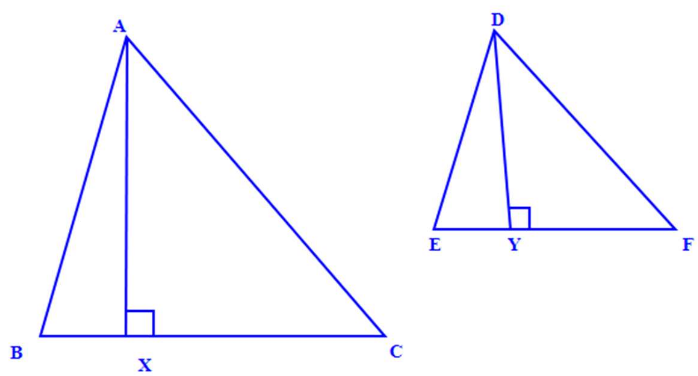
Clearly, $\Delta AXB \sim \Delta CXP$ by the AA criterion.
Thus,

$$\frac{AX}{XC}=\frac{AB}{PC}=\frac{2}{1}$$

$$\Rightarrow \frac{AX}{AX+XC}=\frac{2}{2+1}$$

$$\Rightarrow \frac{AX}{AC}=\frac{2}{3}$$

S34. Consider the following figure



Since $\Delta ABC \sim \Delta DEF$, we have AB: DE = BC: EF. Now, consider ΔABX and ΔDEY. Since $\angle B = \angle E$ and $\angle AXB = \angle DYE = 90^o$, the two triangles are similar by the AA criterion, so that AB : DE = AX : DY. Thus, we conclude that AX : DY = BC : EF.

S35. Make use of the angle bisector theorems in triangles BAC and DAC.

S36. Use the mid-point theorem.

S37. Drop a perpendicular from A onto BC, and use the Pythagoras Theorem.

S38. Use the similarity of triangles ACF and ABD to obtain:

$$\frac{a}{a+b}=\frac{z}{y}$$

Similarly, show that

$$\frac{b}{a+b}=\frac{z}{x}$$

Now, add these two expressions.