Arithmetic Integers

Review Exercise Answers

Level-1

S1. (A). The HCF is the product of the lowest powers of the common factors. In this case, it will be equal to $2^2 \times 3^2 \times 5$

S2. (A). The HCF is the product of the lowest powers of the common factors. In this case, it will be equal to $2^2 \times 5 \times 7^2 = 980$

S3. (C). We have:

$$108 = 2\times2\times3\times3\times3 = 2^2 \times 3^2$$
$$288 = 2\times2\times2\times2\times2\times3\times3 = 2^5 \times 3^2$$
$$360 = 2\times2\times2\times3\times3\times5 = 2^3 \times 3^2 \times 5$$

The HCF is the product of the lowest powers of the common factors. In this case, it will be equal to

$$2^2 \times 3^2 = 36$$

S4. (C). The LCM will be the product of the highest powers of 2, 3, 5, 7 and 11 among these numbers, and hence will be equal to

$$2^3 \times 3^3 \times 5^3 \times 7^4 \times 11$$

S5. (B). We have:

$$72 = 2\times2\times2\times3\times3 = 2^3 \times 3^2$$
$$108 = 2\times2\times3\times3\times3 = 2^2 \times 3^3$$
$$2100 = 2\times2\times3\times5\times5\times7 = 2^2 \times 3 \times 5^2 \times 7$$

The LCM will be the product of the highest powers of 2, 3, 5 and 7 among these numbers, and hence will be equal to

$$2^3 \times 3^3 \times 5^2 \times 7 = 37800$$

S6. (B). The LCM of the three numbers will be the product of the greatest power of each prime factor in the numbers. Therefore, the required LCM is

$$2^{3m} \times 3^{4n} \times 5^{3p} \times 7^{2m+n}$$

S7. (C). We have:

$$\begin{array}{r|l} 3 & 99 \\ \hline 3 & 33 \\ \hline 11 & 11 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 176 \\ \hline 2 & 88 \\ \hline 2 & 44 \\ \hline 2 & 22 \\ \hline 11 & 11 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 182 \\ \hline 7 & 91 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$$\therefore \quad 99 = 1\times3\times3\times11$$
$$101 = 1\times101$$
$$176 = 1\times2\times2\times2\times2\times11$$
$$182 = 1\times2\times7\times13$$

Clearly, 182 has the most number of distinct prime factors: 2, 7 and 13.

S8. (C). Let a and b be the required numbers. We have:

$$a+b=55$$
$$ab = HCF \times LCM = 5\times120 = 600$$

Therefore, the required sum is:

$$\frac{1}{a}+\frac{1}{b}=\frac{b+a}{ab}=\frac{55}{600}=\frac{11}{120}$$

S9. (B). It is given that

$$LCM + HCF = 1260$$
$$LCM - HCF = 900$$

After solving these two equations, we get

$$LCM = 1080,\ HCF = 180$$

Therefore, the product of two numbers is equal to

$$LCM \times HCF = 1080\times180$$
$$= 194400$$

S10. (C), as there is no common factor between consecutive natural numbers.

S11. (D). The LCM of any two prime numbers is their product, since their HCF is 1.

S12. (C). The HCF of all four numbers will be HCF (48, 36), or 12.

S13. (A). The HCF of the given fractions is

$$=\frac{HCF \text{ of } 8, 12, 32}{LCM \text{ of } 21, 35, 7}=\frac{4}{105}$$

S14. (D). The LCM of 5, 15, 25 is 75, and the HCF of 16, 24, 8 is 8. Thus, the LCM of the given fractions is $\frac{75}{8}$

S15. (B). In the other cases, the two numbers have a common factor greater than 1.

S16. (A). We have:

$$-1 = 7(-1) + 6$$
$$-2 = 7(-1) + 5$$
$$-3 = 7(-1) + 4$$
$$-4 = 7(-1) + 3$$

B – MULTIPLE CHOICE

S17. (A), (B), (C) and (D). We will prove that the product of any four consecutive integers will always be a multiple of 24, from which the first three options will automatically be proven as correct. Let us take any four consecutive integers as: $n,\ n+1,\ n+2$ and $n+3$. Note that there will be exactly two even numbers in this set, and one of them will be a multiple of 4. Thus, the product of these numbers will definitely have $4\times 2 = 8$ as a factor. Also, at least one of the four numbers must be a multiple of 3 (there could be two multiples of 3 also, but for us, one will do). This means that the product of the four numbers will also have 3 as a factor. Thus, the product of the four numbers will be a multiple of $8\times 3 = 24$.

Sol. (B) and (C) are incorrect. The rest are correct. Self-exercise.

C – INTEGER ANSWERS

S18. The correct answer is 6. We have:

$$3600 = 2\times 2\times 2\times 2\times 3\times 3\times 5\times 5$$
$$= 2^4\times 3^2\times 5^2$$
$$= 2^4\times 3^2\times 5^2\times 7^0$$

Comparing this with the given prime factorization expression, we get

$$p = 4,\ q = 2,\ l = 0$$
$$\Rightarrow\quad p+q+l = 4+2+0 = 6$$

S19. The correct answer is 210:

$$2\times 3\times 5\times 7 = 210$$

S20. The correct answer is 8. We have:

$$21 = 3\times 7$$

which is the highest common factor of the two given numbers. So, we have:

$$p = 1,\ q = 1$$
$$\Rightarrow\quad 3p+5q = 3(1)+5(1)$$
$$= 8$$

S21. The correct answer is 5. We have:

$$108 = 2\times 2\times 3\times 3\times 3 = 2^2\times 3^3$$

which is the highest common factor of the three given numbers. So, we have:

$$p = 2,\ q = 3$$
$$\Rightarrow\quad p+q = 2+3 = 5$$

S22. We have:

$$24 = 2\times 2\times 2\times 3 = 2^3\times 3$$

which is the highest common factor of the two given numbers. So, we have:

$$p = 1,\ q = 3$$

The two numbers are:

$$2^4 \times 3 \times 5 = 240$$

$$2^3 \times 3^2 \times 43 = 3096$$

Hence, the required sum is 240 + 3096 or 3336.

S23. The correct answer is 702. We have:

$$44676 = 2 \times 2 \times 3 \times 3 \times 17 \times 73$$

$$= 2^2 \times 3^2 \times 17 \times 73$$

$$\Rightarrow \quad p = 2, q = 2, r = 73$$

Thus, the two numbers are

$$2^2 \times 3^2 \times 17 = 612$$

$$2 \times 3^2 \times 73 = 1314$$

Hence, the magnitude of the difference of the numbers is

$$1314 - 612 = 702$$

S24. The correct answer is 111. Let the numbers be 37a and 37b. Then,

$$37a \times 37b = 4107$$

$$\Rightarrow \quad ab = 3$$

Now, the co-primes with the product 3 are (1, 3). So, the required numbers are (37 × 1, 37 × 3) i.e., (37, 111).

Greater number = 111

S25. The correct answer is 40. Let the numbers be 3x, 4x and 5x. Then, their LCM will be 60x.

$$\Rightarrow \quad 60x = 2400 \quad \Rightarrow \quad x = 40$$

The numbers are (3 × 40), (4 × 40) and (5 × 40). Hence, their HCF will be 40.

S26. The correct answer is 2. Let the numbers be 13a and 13b. We have:

$$13a \times 13b = 2028$$

$$\Rightarrow \quad ab = 12$$

Now, the coprimes with product 12 are (1, 12) and (3, 4). So, the required numbers are (13 × 1, 13 × 12) and (13 × 3, 13 × 4). Clearly, there are 2 such pairs.

S27. The correct answer is 308. We know that the product of two numbers is equal to the product of their LCM and HCF. Therefore, the other number will be equal to

$$\frac{11 \times 7700}{275} = 308$$

S28. The correct answer is 48. Let the numbers be 3x and 4x. Then, their HCF will be x. So, x = 4, and the numbers are 12 and 16.

LCM (12, 16) = 48

S29. The correct answer is 40. Let the numbers be $2x$ and $3x$. Then, their LCM will be $6x$. So,

$$6x = 48 \Rightarrow x = 8$$

Thus, the numbers are 16 and 24 and the required sum is $16 + 24 = 40$.

S30. Using the relation HCF (*p*, *q*) × LCM (*p*, *q*) = *p* × *q*, we get

$$x = \frac{LCM(x,18)\,HCF(x,18)}{18}$$

$$= \frac{36\times 2}{18} = 4$$

S31. The correct answer is 24. Clearly, *p* divides (248 – 8), i.e., 240 and (1032 – 8), i.e., 1024. So, the required number is HCF (240, 1024). We have:

$$240 = 2\times2\times2\times2\times3\times5$$

$$= 2^4\times3\times5$$

$$1032 = 2\times2\times2\times3\times42$$

$$= 2^3\times3\times43$$

$$\Rightarrow HCF(240,1032) = 2^3\times3 = 24$$

Hence, the required number is 24.

S32. The correct answer is 108. Clearly, *p* divides (546 – 6), i.e., 540 and (764 – 8), i.e., 756. So, the required number is HCF (540, 756). We have:

$$540 = 2\times2\times3\times3\times3\times5$$

$$= 2^2\times3^3\times5$$

$$756 = 2\times2\times3\times3\times3\times7$$

$$= 2^2\times3^3\times7$$

$$\Rightarrow HCF(540,756) = 2^2\times3^3$$

$$= 4\times27 = 108$$

Hence, the required number is 108.

S33. The correct answer is 21. Resolving 504 and 735 into prime factors, we have:

$$504 = 2\times2\times2\times3\times3\times7 = 2^3\times3^2\times7$$

$$735 = 3\times5\times7\times7 = 3\times5\times7^2$$

The HCF of the two numbers is 3 × 7 = 21. Therefore, the capacity of the required container is 21 litres.

S34. We determine the prime factorization of 8640:

$$8640 = 2^6\times3^3\times5$$

The powers of 2 and 3 are 6 and 3 respectively, and hence the required sum is 6 + 3 = 9.

S35. We determine the prime factorization of these two numbers:

$$1120 = 2^5\times5^1\times7^1$$

$$1512 = 2^3\times3^3\times7^1$$

Looking at the powers of the various prime factors, the HCF can now be written as:

$$\text{HCF} = 2^3\times7^1 = 56$$

S36. We determine the prime factorization of these two numbers:

$$225 = 3^2 \times 5^2$$
$$280 = 2^3 \times 5^1 \times 7^1$$

Looking at the powers of the various prime factors, the LCM can now be written as:

$$\text{LCM} = 2^3 \times 3^2 \times 5^2 \times 7^1 = 12600$$

S37. We have:

$$60 = 2^2 \times 3^1 \times 5^1$$
$$80 = 2^4 \times 5^1$$
$$110 = 2^1 \times 5^1 \times 11^1$$
$$\Rightarrow \quad \text{LCM}(60, 80) = 2^4 \times 3^1 \times 5^1 = 240$$
$$\Rightarrow \quad \text{LCM}(60, 80, 110) = \text{LCM}(240, 110)$$
$$= 2^4 \times 3^1 \times 5^1 \times 11^1$$
$$= 2640$$

S38. We have:

$$120 = 2^3 \times 3^1 \times 5^1$$
$$150 = 2^1 \times 3^1 \times 5^2$$
$$180 = 2^2 \times 3^2 \times 5^1$$
$$\Rightarrow \text{LCM}(120,150) = 2^3 \times 3^1 \times 5^2$$
$$= 600$$
$$\Rightarrow \text{LCM}(600,180) = 2^3 \times 3^2 \times 5^2$$
$$= 1800$$

S39. We have:

$$10 = 2^1 \times 5^1, \ \ 256 = 2^8$$

Clearly, the minimum power to which 10 must be raised so that we obtain a multiple of 256,
is 8:

$$10^8 = \left(2^1 \times 5^1\right)^8 = 2^8 \times 5^8 = 256 \times 5^8$$

S40. We have:

$$21 = 3^1 \times 7^1, \ \ 343 = 7^3$$

Clearly, the minimum power to which 21 must be raised so that we obtain a multiple of 343,
is 3:

$$21^3 = \left(3^1 \times 7^1\right)^3 = 3^3 \times 7^3 = 343 \times \left(3^3\right)$$

S41. We have:

$$6 = 2^1 \times 3^1,\ \ 729 = 3^6$$

Clearly, the minimum power to which 6 must be raised so that we obtain a multiple of 729, is 6:

$$6^6 = \left(2^1 \times 3^1\right)^6 = 2^6 \times 3^6 = 729 \times \left(2^6\right)$$

S42. Let the smallest such integer be n. We have:

$$n = 4p + 3 = 11q + 10$$
$$\Rightarrow\ 11q = 4p - 7 = 4p - 8 + 1 = 4r + 1$$

Thus, we have to think of the smallest positive multiple of 11 which is 1 more than a multiple of 4. This is 33, and so:

$$11q = 33\ \Rightarrow\ q = 3$$

Thus, $n = 11(3) + 10 = 43$.

S43. Let the smallest such integer be n. We have:

$$n = 13p + 10 = 17q + 10$$
$$\Rightarrow\ 13p = 17q$$

The smallest positive pair of integers which satisfy this will be $p = 17$ and $q = 13$. Thus,

$$n = 13 \times 17 + 10 = 221 + 10 = 231$$

S44. We can write the number as $n = 3k + 1$. Thus,

$$n^2 = (3k+1)^2 = 9k^2 + 6k + 1$$
$$= 3\left(3k^2 + 2k\right) + 1$$

Clearly, when n^2 is divided by 3, the remainder will again be 1.

S45. Let n be an odd integer, so that we can write $n = 2m + 1$. Now,

$$n^2 = (2m+1)^2 = 4m^2 + 4m + 1$$
$$= 4\left(m^2 + m\right) + 1$$

When this is divided by 4, the only possible remainder is 1. Thus, the final answer is also 1.

S46. We can write $n = 5k + 1$, and so:

$$n^2 = (5k+1)^2 = 25k^2 + 10k + 1$$
$$= 5k(5k+2) + 1$$

Now, two cases arise.

Case-1: *k* is even.
We let $k = 2m$, so that

$$n = 5(2m)(10m+2) + 1$$
$$= 10(m(10m+2)) + 1$$

Clearly, this when divided by 10 will leave a remainder of 1.

Case-2: *k* is odd.
We let $k = 2m + 1$, so that

$$n = 5(2m+1)(10m+5+2) + 1$$
$$\Rightarrow \quad n = 5(2m+1)(10m+7) + 1$$
$$\Rightarrow \quad n = 5(20m^2 + 24m + 7) + 1$$

Thus,

$$n = 5(20m^2 + 24m) + 36$$
$$= 10q + 6 \ \text{(how ?)}$$

This when divided by 10 will leave the remainder 6. We see that there are two possible remainders: 1 and 6. Thus, the required sum is 7.

S47. We have:

$$8448 = 5082(1) + 3366 \qquad [\text{Step-1}]$$
$$5082 = 3366(1) + 1716 \qquad [\text{Step-2}]$$
$$3366 = 1716(1) + 1650 \qquad [\text{Step-3}]$$
$$1716 = 1650(1) + 66 \qquad [\text{Step-4}]$$
$$\boxed{1650} = \underline{66} \quad (25) + 0 \qquad [\text{Step-5}]$$
$$\Rightarrow \quad HCF(8448, 5082) = 66$$

Thus, the dividend in the last step is 1650.

S48. We have:

$$6699 = 5655(1) + 1044 \quad [\text{Step-1}]$$

$$5655 = 1044(5) + 435 \quad [\text{Step-2}]$$

$$1044 = 435(2) + 174 \quad [\text{Step-3}]$$

$$435 = 174(2) + 87 \quad [\text{Step-4}]$$

$$\boxed{174} = \underline{87}(2) + 0 \quad [\text{Step-5}]$$

$$\Rightarrow \; HCF(6699, 5655) = 87$$

Thus, the dividend in the last step is 174.

S49. The answer is 9, because:

$$-13 = 11 \times (-2) + 9$$

Recall that the remainder is always positive.

S50. We have:

$$\text{Step - 1: } 7854 = 4746(1) + 3108$$

$$\text{Step - 2: } 4746 = 3108(1) + 1638$$

$$\text{Step - 3: } 3108 = 1638(1) + 1470$$

$$\text{Step - 4: } 1638 = 1470(1) + 168$$

$$\text{Step - 5: } 1470 = 168(8) + 126$$

$$\text{Step - 6: } 168 = 126(1) + 42$$

$$\text{Step - 7: } 126 = \underline{42}(3) + 0$$

$$\Rightarrow \; \text{HCF}(7854, 4746) = 42$$

The algorithm terminates in 7 steps.

D – MISCELLANEOUS

S51. $H(101)$, because its denominator will be smaller than $H(100)$, as it has only two divisors (it is a prime number).

S52. The given statement is false. Consider these two pairs of numbers: (1,40) and (5,8). The HCF of both pairs is 1, and the LCM of both pairs is 40. Thus, we have demonstrated two different pairs with the same values of HCF and LCM. Clearly, given the HCF and LCM of two numbers, they cannot be determined uniquely. Another example is the following two pairs: (5,18) and (9,10). The HCF of both pairs is 1, and the LCM of both is 90.

S53. The given statement is false. An odd integer, upon division by 4, can leave a remainder of 1 or 3. Therefore, it can be written as either

$4k + 1$ or $4k + 3$.

S54. This should intuitively be obvious, but let us prove it rigorously. Let n, $n+1$ and $n+2$ be any three consecutive integers. If n is a multiple of 3, then we are done. If n is not a multiple of 3, then two cases arise:

Case-1: $n = 3k + 1$
In this case, $n+2$ will be a multiple of 3, because

$$n+2=3k+1+2=3k+3=3(k+1).$$

Case-2: $n = 3k + 2$
In this case, $n+1$ will be a multiple of 3, because

$$n+1=3k+2+1=3k+3=3(k+1).$$

Clearly, in any three consecutive integers, one will always be a multiple of 3.

S55. The given statement is true. Let $2n+1$, $2n+3$ and $2n+5$ be three odd consecutive integers. If $2n+1$ is an odd multiple of 3, then we are done. But suppose that it is not, and so two cases arise:

Case-1: $2n + 1 = 3k + 1$, k is even.
k is even because both sides must be odd. In this case, $2n+3$ will be a multiple of 3, because

$$2n+3=3k+1+2=3k+3=3(k+1)$$

And since k is even, $k+1$ is odd, so $3(k+1)$ is an odd multiple of 3.

Case-2: $2n + 1 = 3k + 2$, k is odd.
Note that k is odd in this case because both sides must be odd. In this case, $2n+5$ will be an odd multiple of 3, because

$$2n+5=3k+2+4=3k+6=3(k+2)$$

And since k is odd, $k+2$ is odd, so $3(k+2)$ is an odd multiple of 3.

S56. The given statement is false. In any three consecutive even integers, one will always be a multiple of 6. Let's see how. Let $2n$, $2n+2$ and $2n+4$ be three consecutive even integers. If $2n$ is a multiple of 6, we are done, but let us suppose that is not. Two cases now arise:

Case-1: $2n = 6k + 2$.
In this case, $2n+4$ will be a multiple of 6.

Case-1: $2n = 6k + 4$.
In this case, $2n+2$ will be a multiple of 6.

You can convince yourself that no other cases are possible. Thus, in any three consecutive even integers, one will always be a multiple of 6.

S57. The given statement is true. Let $2n,\ 2n+2,\ 2n+4$ and $2n+6$ be four consecutive even integers. If $2n$ is a multiple of 8, then we are done, and you can easily show that none of the other three will be a multiple of 8. Now, suppose that $2n$ is not a multiple of 8. Three cases arise:

Case-1: $2n = 8k + 2$.
In this case, $2n+6$ will be a multiple of 8.

Case-1: $2n = 8k + 4$.
In this case, $2n+4$ will be a multiple of 8.

Case-1: $2n = 8k + 6$.
In this case, $2n + 2$ will be a multiple of 8.

This exhausts all possible cases.

S58. The given statement is false. Here is a counter-example: 7, 9, 11 and 13. These are four consecutive odd integers and none is a multiple of 5. However, if we take five consecutive odd integers, exactly one of them will be a multiple of 5.

S59. The given statement is false. A third case is possible: $3k+2$.

S60. The given statement is true. Let the three consecutive integers be $n,\ n+1$ and $n+2$. Note that at least one (and maybe two) of these will be even, and exactly one of these will be a multiple of 3. Thus, when you multiply the three integers, the resulting product will have both 2 and 3 as factors, and so it will be a multiple of 6.

S61. We let $n = 2m+1$. Now,

$$n^2 - 1 = (n+1)(n-1)$$
$$= (2m+1+1)(2m+1-1)$$
$$\Rightarrow n = 2m(2m+2) = 4m(m+1)$$

We note that $m(m+1)$ will always be even (why?), and so, n will always be a multiple of $4 \times 2 = 8$.

S62. Let n be an odd multiple of 3. Then, we can write $n = 3m$, where m is odd. We let $m = 2p+1$. Now, $n = 3(2p+1) = 6p+3$.

Case-1: p is odd.
We let $p = 2q+1$ and so:

$$n = 6(2q+1) + 3 = 12q + 9$$
$$= 4(3q+2) + 1 = 4k + 1$$

Case-2: p is even.
We let $p = 2q$ and so:

$$\begin{aligned} n &= 6(2q) + 3 = 12q + 3 \\ &= 4(3q+1) - 1 = 4k - 1 \end{aligned}$$

Thus, n can be expressed as either $4k+1$ or $4k-1$.

S63. (a) Let $(a,b) = d$. We recall that d is the least possible positive linear combination of a and b. That is, d is the least possible positive value of $ax + by$.
Now, (ma, mb) will be the least possible positive linear combination of ma and mb. That is,

$$\begin{aligned} (ma, mb) &= \min\{(ma)x + (mb)y\} \\ &= m \times \min\{ax + by\} = md \end{aligned}$$

(b) We use the result of part (a) in a slightly modified version: $m(a,b) = (ma, mb)$. Thus,

$$d \times \left(\frac{a}{d}, \frac{b}{d}\right) = \left(d \times \frac{a}{d}, d \times \frac{b}{d}\right) = (a,b)$$
$$\Rightarrow \left(\frac{a}{d}, \frac{b}{d}\right) = \frac{(a,b)}{d}$$

S64. From a prime factorization point of view, a shares no common prime factor with n, and neither does b. Thus, ab will also not share any prime factor with n, which means that ab and n are co-prime.

You are urged to try an alternate proof on the following lines. Since $(a,n) = 1$, a linear combination of a and n will be 1; similarly, for b and n. Now, try to show that a linear combination of ab and n can be 1 – this will mean that $(ab, n) = 1$.

S65. Note that this is possible because $(30, 41) = 1$. We apply Euclid's Division Algorithm to 30 and 41:

$$\begin{aligned} 41 &= 30(1) + 11 \\ 30 &= 11(2) + 8 \\ 11 &= 8(1) + 3 \\ 8 &= 3(2) + 2 \\ 3 &= 2(1) + 1 \\ 2 &= \underline{1}(2) + 0 \end{aligned}$$

Now, we backtrack and express the GCD 1 as a linear combination of 30 and 41:

$$\begin{aligned} 1 &= 3(1)-2(1)=3(1)-(8-3(2))(1) \\ &= 3(3)-8(1)=(11(1)-8(1))(3)-8(1) \\ &= 11(3)-8(4)=11(3)-(30(1)-11(2))(4) \\ &= 11(11)-30(4)=(41(1)-30(1))(11)-30(4) \\ &= 41(11)-30(15) \end{aligned}$$

Thus,

$$30(-15)-41(-11)=1$$
$$\Rightarrow\ a=-15,\ b=-11$$

S66. We note that both m and n are even numbers which are not multiples of 4. Thus, both are of the form $4p+2$, so there sum will be of the form $4k'+4$ or $4k$, that is, a multiple of 4. Thus, $(m+n,4)$ will equal 4.

S67. Assume that m and n are positive (m is greater). Now,

$$(m+n)(m-n)=p$$

This leads to (think why):

$$m+n=p,\ m-n=1$$
$$\Rightarrow\ m=\frac{p+1}{2},\ n=\frac{p-1}{2}$$

The negative of these values will also form valid solutions.

S68. We only count the number of 5's, since the number of 2's will be larger. The number of 5's will be:

$$\left[\frac{1000}{5}\right]+\left[\frac{1000}{25}\right]+\left[\frac{1000}{125}\right]+\left[\frac{1000}{625}\right]$$
$$=200+40+8+1=249$$

Thus, there will be 249 zeroes at the end of 1000!.

S69. 2, 6 and 8 are factors, while 5 and 9 are not.

S70. Self-exercise.

S71. Self-exercise.